CORE
Metadata, citation and similar papers at core.ac.uk
Provided by KITopen

ISSN 2072-5981

Magnetic Resonance in Solids

Electronic Journal

Volume ***16***
Issue ***3***
Paper No ***14301***
1-5 pages
2014

http://mrsej.kpfu.ru
http://mrsej.ksu.ru

Established and published by Kazan University
Sponsored by International Society of Magnetic Resonance (ISMAR)
Registered by Russian Federation Committee on Press, August 2, 1996
First Issue was appeared at July 25, 1997

© Kazan Federal University (KFU)*

"***Magnetic Resonance in Solids. Electronic Journal***" (**MRS*ej***) is a peer-reviewed, all electronic journal, publishing articles which meet the highest standards of scientific quality in the field of basic research of a magnetic resonance in solids and related phenomena. **MRS*ej*** is free for the authors (no page charges) as well as for the readers (no subscription fee). The language of **MRS*ej*** is English. All exchanges of information will take place via Internet. Articles are submitted in electronic form and the refereeing process uses electronic mail. All accepted articles are immediately published by being made publicly available by Internet (http://MRSej.kpfu.ru).

Editors-in-Chief
Jean **Jeener** (Universite Libre de Bruxelles, Brussels)
Boris **Kochelaev** (KFU, Kazan)
Raymond **Orbach** (University of California, Riverside)

Executive Editor
Yurii **Proshin** (KFU, Kazan)
mrsej@kpfu.ru
editor@ksu.ru

Editors
Vadim **Atsarkin** (Institute of Radio Engineering and Electronics, Moscow)
Yurij **Bunkov** (CNRS, Grenoble)
Mikhail **Eremin** (KFU, Kazan)
David **Fushman** (University of Maryland, College Park)
Hugo **Keller** (University of Zürich, Zürich)
Yoshio **Kitaoka** (Osaka University, Osaka)
Boris **Malkin** (KFU, Kazan)
Alexander **Shengelaya** (Tbilisi State University, Tbilisi)
Jörg **Sichelschmidt** (Max Planck Institute for Chemical Physics of Solids, Dresden)
Haruhiko **Suzuki** (Kanazawa University, Kanazava)
Murat **Tagirov** (KFU, Kazan)
Dmitrii **Tayurskii** (KFU, Kazan)

* In Kazan University the Electron Paramagnetic Resonance (EPR) was discovered by Zavoisky E.K. in 1944.

Alternative theoretical approach of the exact ferromagnetic resonance frequency for common sample shapes by considering the phenomenological damping parameter – A tribute to C. Kittel and T. L. Gilbert

K. Seemann*, H. Leiste, K. Krüger

Karlsruhe Institute of Technology KIT (Campus North), Institute for Applied Materials, Hermann von Helmholtz Platz 1, 76344 Eggenstein-Leopoldshafen, Germany

**E-mail: klaus.seemann@kit.edu*

(Received June 16, 2014; accepted July 25, 2014)

In order to establish an expression for the ferromagnetic resonance frequency of specimen with general shape, the Landau-Lifschitz-Gilbert differential equation was solved by considering demagnetisation effects and damping. By beginning the approach of calculation in a three dimensional space the sample was regarded as a uniformly magnetised ferro- or ferrimagnetic conventional ellipsoid. The condition for calculation must be a one-domain state which can be obtained by introducing a marked uniaxial anisotropy field and/or by applying an external magnetic saturation field. This can now be put on the level with a macro-spin precessing about its preferred direction. As a result, a more exact solution for the ferromagnetic resonance frequency was achieved which takes the phenomenological damping parameter as well as the demagnetisation factors into account. Applying it to certain sample shapes (plane, spherical or cylindrical) the uniaxial anisotropy field, the saturation magnetisation and especially the damping parameter show a different impact on the resonance frequency value. It could be shown that a plane sample (film) is more influenced by the damping parameter.

PACS: 75.30.Gw, 75.40.Gb, 76.20.+q, 76.50.+g

Keywords: Landau-Lifschitz-Gilbert differential equation, FMR, damping parameter, demagnetisation factors

1. Introduction

In 1946, the ferromagnetic resonance (FMR) absorption was discovered by J. H. E. Griffiths [1]. The interaction of ferromagnetic solid state with alternating magnetic fields was thereupon theoretically described by C. Kittel [2]. In his work, he showed that there is a deviation of the ferromagnetic resonance frequency to the simple Larmor frequency of electrons with spin 1/2 in a homogeneous static magnetic field H, although, ferromagnetism in transition metals is determined by the electron spin. In measurements carried out by Griffiths, plane samples were employed which explained the difference between the frequencies. By introducing an effective magnetic field namely a "fictitious" field $(B{\cdot}H)^{1/2}$, where B is the induction, this anomalous behaviour could be explained. In a further work, Kittel showed a general description of the "lossless" ferromagnetic resonance frequency for an effective magnetic field which arises inside a general specimen underlying the demagnetisation effect [3]. At this point, we would like to settle the question how the "real" ferromagnetic resonance frequency in lossy samples behaves where damping plays an essential role. The Landau-Lifschitz-Gilbert linear differential equation [4], which describes the motion of magnetic moments in an effective magnetic field, lends itself to calculate the lossy ferromagnetic resonance frequency. As a vector equation it allows to regard and apply an anisotropic material behaviour. In the following section we intend to derive a general ferromagnetic resonance frequency formula by solving the Landau-Lifschitz-Gilbert equation employing an analytical solution method for differential equations.

2. Quasi-classical approach of the exact ferromagnetic resonance frequency absorption

In order to theoretically describe the ferromagnetic resonance frequency f_{FMR} at which any ferromagnetic sample absorbs the maximum magnetic high-frequency energy, C. Kittel [3] referred to the simple lossless form of the Landau-Lifschitz equation [5]. This was an excellent approach if one considers damping as a negligible effect, and in quantum mechanics one obtains the same result for the resonance condition. But in reality and under certain conditions damping cannot be ignored which has already been proven in [6]. At this point and in order to obtain a more universal description of the FMR absorption, we try to make an approach by means of the well-established Landau-Lifschitz-Gilbert (LLG) linear differential equation

$$\frac{\partial \vec{M}}{\partial t} = -\gamma \vec{M} \times \vec{H}_{\text{eff}} + \frac{\alpha}{M_s}\left(\vec{M} \times \frac{\partial \vec{M}}{\partial t}\right). \tag{1}$$

For convenience, eddy-currents are neglected, i.e., in the case of a ferro- or ferrimagnetic film, it must be thin enough and/or has to possess a sufficiently high resistivity [7]. In the case of bulk materials a high resistivity should be present. Finally, a minimised magneto-crystalline anisotropy, obtained by the material composition and the amorphous or nano-crystalline material state, is assumed for calculation. The essence of equation (1) is the effective magnetic field vector H_{eff} inside the sample which obviously determines the precession of its total magnetic moment density $J = M/\gamma$, where γ is the gyromagnetic constant. Assuming that the effective magnetic field possesses the main macroscopic components H_{u}, which is a field caused by the uniaxial anisotropy field, the high-frequency field h and the demagnetisation field H_{d}, we can write the effective magnetic field vector in the following form

$$\vec{H}_{\text{eff}} = \vec{H}_{\text{u}} + \vec{h} + \vec{H}_{\text{d}} = \begin{pmatrix} 0 \\ 0 \\ H_{\text{u}} \end{pmatrix} + \begin{pmatrix} h_x \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -N_x \cdot m_x \\ -N_y \cdot m_y \\ -N_z \cdot M_s \end{pmatrix} = \begin{pmatrix} h_x - N_x m_x \\ -N_y m_y \\ H_{\text{u}} - N_z M_s \end{pmatrix}. \tag{2}$$

The matrix elements which are the demagnetisation factors N_x, N_y and N_z in the diagonal demagnetization matrix N have to fulfil the requirement, $\operatorname{Tr} N = 1$. If the exiting high-frequency field is sufficiently small it can be neglected for further calculations. By putting (2) into (1) LLG results in

$$\begin{pmatrix} \frac{\partial m_x}{\partial t} \\ \frac{\partial m_y}{\partial t} \\ \frac{\partial m_z}{\partial t} \end{pmatrix} = -\gamma \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} \times \begin{pmatrix} -N_x m_x \\ -N_y m_y \\ H_u - N_z M_s \end{pmatrix} + \frac{\alpha}{M_s} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} \times \begin{pmatrix} \frac{\partial m_x}{\partial t} \\ \frac{\partial m_y}{\partial t} \\ \frac{\partial m_z}{\partial t} \end{pmatrix}. \tag{3}$$

For small amplitudes of the magnetic moment ($h \to 0$) it now makes sense to assume $m_z \sim M_s$, if the material is uniformly magnetised in the z-direction by H_{u} and/or by an external magnetic field. Calculating (3) and eliminating $\partial m_x / \partial t$ and $\partial m_y / \partial t$ in the x- and y-direction, respectively, leads to a set of coupled linear homogeneous differential equations

$$\begin{aligned} \frac{\partial m_x}{\partial t} &= -\gamma\alpha \frac{H_{\text{u}} - M_s\left(N_z - N_x\right)}{1+\alpha^2} m_x - \gamma \frac{H_{\text{u}} - M_s\left(N_z - N_y\right)}{1+\alpha^2} m_y, \\ \frac{\partial m_y}{\partial t} &= \gamma \frac{H_{\text{u}} - M_s\left(N_z - N_x\right)}{1+\alpha^2} m_x - \gamma\alpha \frac{H_{\text{u}} - M_s\left(N_z - N_y\right)}{1+\alpha^2} m_y, \\ \frac{\partial m_z}{\partial t} &\approx 0. \end{aligned} \tag{4}$$

In order to determine the ferromagnetic resonance frequency, we determine the coefficient of the time-dependent exponent of the general solution for the first two equations in (4) by solving the determinant

$$\det(A-\lambda I)=\begin{vmatrix} -\gamma\alpha\dfrac{H_\mathrm{u}-M_s\left(N_z-N_x\right)}{1+\alpha^2}-\lambda & -\gamma\dfrac{H_\mathrm{u}-M_s\left(N_z-N_y\right)}{1+\alpha^2} \\ \gamma\dfrac{H_\mathrm{u}-M_s\left(N_z-N_x\right)}{1+\alpha^2} & -\gamma\alpha\dfrac{H_u-M_s\left(N_z-N_y\right)}{1+\alpha^2}-\lambda \end{vmatrix}=0\,. \tag{5}$$

As a result, one obtains a binomial equation

$$\lambda_{1/2}=-\frac{1}{2}\gamma\alpha\frac{2H_\mathrm{u}-M_s\left(2N_z-N_x-N_y\right)}{1+\alpha^2}\pm \\ \pm\frac{\gamma}{1+\alpha^2}\sqrt{\alpha^2\frac{M_s^2\left(N_x-N_y\right)^2}{4}-H_\mathrm{u}^2-H_\mathrm{u}M_s\left(N_x+N_y-2N_z\right)-M_s^2\left(N_z-N_x\right)\left(N_z-N_y\right)}\,, \tag{6}$$

which generates two conjugate complex solutions for a damping constant $\alpha < 1$. But equation (6) can then be converted into the more usable form

$$\lambda_{1/2}=\frac{1}{2}\gamma\alpha\frac{M_s\left(2N_z-N_x-N_y\right)-2H_\mathrm{u}}{1+\alpha^2}\pm \\ \pm i\frac{\gamma}{1+\alpha^2}\sqrt{-\alpha^2\frac{M_s^2\left(N_x-N_y\right)^2}{4}+H_\mathrm{u}^2+H_\mathrm{u}M_s\left(N_x+N_y-2\cdot N_z\right)+M_s^2\left(N_z-N_x\right)\left(N_z-N_y\right)}\,, \tag{7}$$

for which the discriminant represents the ferromagnetic resonance frequency condition for samples of general shape. It is exhibited in SI units as follows

$$f_\mathrm{FMR}=\frac{\gamma\mu_0}{2\pi\left(1+\alpha^2\right)}\sqrt{H_\mathrm{u}^2+H_\mathrm{u}M_s\left(N_x+N_y-2N_z\right)+M_s^2\left(N_z-N_x\right)\left(N_z-N_y\right)-\alpha^2\frac{M_s^2\left(N_x-N_y\right)^2}{4}} \tag{8}$$

and generically describes the ferromagnetic resonance frequency already obtained by [8] in an absolutely different way. From formula (8) we like to deviate the ferromagnetic resonance frequency for certain types of sample shapes which are predestined and often used for FMR experiments.

I. Plane sample with lateral dimensions much larger than its thickness [6], $N_x = N_z = 0$, $N_y = 1$:

$$f_\mathrm{FMR}=\frac{\gamma}{2\pi\left(1+\alpha^2\right)}\mu_0\sqrt{H_\mathrm{u}^2+H_\mathrm{u}M_s-\alpha^2\frac{M_s^2}{4}}\,. \tag{9}$$

II. Spherical sample, $N_x = N_y = N_z = 1/3$:

$$f_\mathrm{FMR}=\frac{\gamma}{2\pi\left(1+\alpha^2\right)}\mu_0 H_\mathrm{u}\,. \tag{10}$$

III. Very long circular cylinder with radius much smaller than its length, $N_x = N_y = 1/2$, $N_z = 0$:

$$f_\mathrm{FMR}=\frac{\gamma}{2\pi\left(1+\alpha^2\right)}\mu_0\sqrt{H_\mathrm{u}^2+H_\mathrm{u}M_s+\frac{M_s^2}{4}}=\frac{\gamma}{2\pi\left(1+\alpha^2\right)}\mu_0\left(H_\mathrm{u}+\frac{M_s}{2}\right). \tag{11}$$

If an external magnetic field in the *z*-direction is applied it has to be added to the uniaxial anisotropy field H_u or can be substituted if the uniaxial anisotropy is not present. The following consideration and discussion are confined to H_u and α only.

3. Results and discussion

By solving the LLG the ensemble of magnetic moments are considered as a magnetic macro-spin precessing about the direction at which it is oriented due to the uniaxial anisotropy field. This approach can be made without considering the microscopic ferromagnetic exchange field within the exchange length of a few nanometres, although, it is assumed parallel to the saturation magnetisation M_s. It is clear that it does not affect the spin precession within such a length scale because in the first place, it only causes the spontaneous magnetisation but not its preferred direction. For various sample shapes it is conspicuous that the uniaxial anisotropy H_u and the damping parameter α generate a different impact on the resonance frequency. If the uniaxial anisotropy is zero, e.g., (9) becomes imaginary and the equation does not bare any physical solution, except for $\alpha = 0$ for which $f_{FMR} = 0$. Overall real solutions are predicted by the equations (10) and (11). For spherical samples f_{FMR} vanishes if $H_u = 0$. If $\alpha = 0$ the resonance formula (10) reflects the pure Larmor frequency equation $\omega_0 = \gamma H_u$. Regarding the last case (III), the cylindrical sample shape, one can easily notice that there is a finite ferromagnetic resonance frequency for zero uniaxial anisotropy. This is evident due to the magnetic spin which is compelled into the z-direction by the strong demagnetisation effect, represented by the factors N_x and N_y. By it, a "demagnetization" anisotropy field along the remaining z-direction is automatically generated. In Fig. 1, the theoretical resonance conditions (I–III) dependent on H_u are depicted for a damping parameter $\alpha = 0.01$ and a saturation polarization $J_s = \mu_0 \cdot M_s = 1.4\ T$.

A gyromagnetic constant γ whose value was set to around 190 GHz/T and used for computation provides a g-factor of approximately 2 for spin-1/2 magnetism. The clearest non-linear change in f_{FMR} on H_u can be seen for plane samples whereas a long cylinder remains insignificantly affected for small anisotropy fields. In spheres, f_{FMR} linearly increases with H_u but shows the smallest ferromagnetic resonance frequency increase. It can be observed that damping in plane samples is more pronounced than in spherical and cylindrical specimen, which pushes f_{FMR} to lower frequency values (Fig. 2). The damping parameter α marginally diminishes the ferromagnetic resonance frequency for the latter only. This can be obviously interpreted by means of the demagnetization factors N_x and N_y. If $N_x \neq N_y$ they cause high demagnetization field, which slightly drives the magnetic moments in a more

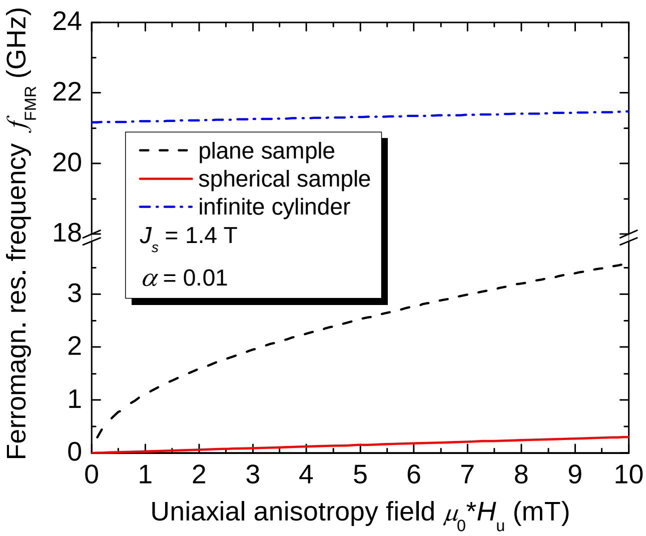

Figure 1. Ferromagnetic resonance frequency dependent on the uniaxial anisotropy field H_u for different sample geometries.

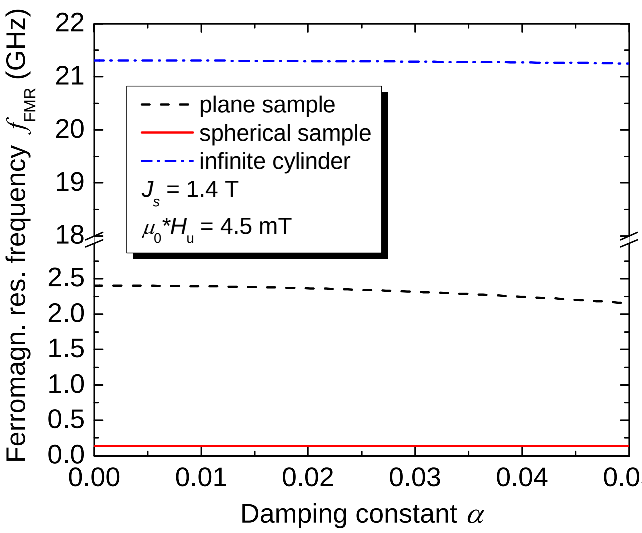

Figure 2. Ferromagnetic resonance frequency dependent on the damping parameter α for different sample geometries.

dissipative precession with wave vector $k \sim 0$. This results in dissipation of energy due to a high perturbation from the strong demagnetisation in the y-direction which affects intrinsic damping mechanisms. In the case of $N_x = N_y$, a weaker demagnetization field does not extensively perturb the $k = 0$ precession for which the last term in the root of expression (8) disappears. Any additional impact of damping can now be excluded.

4. Conclusions

In the present paper, we have introduced a theoretic model for the ferromagnetic resonance frequency which includes the expression for damping by solving the quasi-classical Landau-Lifschitz-Gilbert differential equation of motion. This opens the access to an alternative approach of the ferromagnetic resonance frequency behaviour dependent on the damping parameter α which represents intrinsic and extrinsic precession damping mechanisms. The origin of these damping mechanisms has already been elaborately discussed, elsewhere. The all comprising and exact formula for f_{FMR} now shows that it can be used for almost arbitrarily shaped samples. The most common specimen concretely treated above exhibit a representative cross section of samples often used in FMR experiments. Despite the "lossless" ferromagnetic resonance frequencies for these sample shapes are well known, the impact of damping shows, although quite small but measurable, some remarkable features. It could be demonstrated that the influence of damping can possess a different significance due to demagnetization effects.

Concerning certain applications the frequency response imposed by the material and its shape should be known, which is an important magnitude for designing high-frequency devices like micro-inductors and transformers with cores, magnetic sensors, magnetic storage components etc., and even for EMC issues.

Acknowledgement

This work was partially supported by the Karlsruhe Nano Micro Facility (KNMF, www.knmf.kit.edu), a Helmholtz research infrastructure at the Karlsruhe Institute of Technology (KIT, www.kit.edu).

References

1. Griffiths J.H.E. *Nature* **158**, 670 (1946)
2. Kittel C. *Phys. Rev.* **71**, 270 (1947)
3. Kittel C. *Phys. Rev.* **73**, 155 (1948)
4. Gilbert T.L. *IEEE Trans. Magn.* **40**, 3443 (2004)
5. Landau L., Lifshitz E. *Physikalische Zeitschriften Sowjet Union* **8**, 153 (1935)
6. Seemann K., Leiste H., Krüger K. *J. Magn. Magn. Mater.* **345**, 36 (2013)
7. Seemann K., Leiste H., Bekker V. *J. Magn. Magn. Mater.* **278**, 200 (2004)
8. Gurevich A.G., Melkov G.A. *Magnetization Oscillations and Waves*, *CRC Press*, (1996)