Impact of systematic errors in gravity and heights on a quasi-geoid model for the Netherlands and Belgium

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Abstract
In this study, we quantified systematic errors in surface gravity anomalies, which were caused by systematic errors in gravity and heights of the gravity stations, and computed their impact on the quasi-geoid model of the Netherlands and Belgium. We found that 70% of the gravity datasets have statistically significant biases ranging from -2 mGal to 1.5 mGal. The primary impact of the biases are long-wavelength systematic distortions in the quasi-geoid model with a peak-to-peak amplitude of 8 cm. We also found systematic errors in the height networks of the Netherlands and Belgium, which cause corresponding errors in the heights of the gravity stations. They range from -3.0 cm to 1.7 cm and -12.0 cm to 5.0 cm, respectively. They also introduce errors in the transformation parameters to EVRF2007 of several centimetres. However, the impact of the height errors on the quasi-geoid model is negligible with a peak-to-peak amplitude of less than 0.1 cm.

1 Introduction

Traditionally, spirit levelling is the primary geodetic measurement technique for measuring height differences between stations. Using spirit levelling to determine heights, requires a network of bench marks (BMs) with known heights, which is maintained by governmental agencies (e.g., Rijkswaterstaat in the Netherlands and the National Geographical Institute in Belgium). The heights of the BMs are determined using precise spirit levelling with or without gravity corrections. Usually, they are defined with respect to a national datum, such as the Normaal Amsterdams Peil (NAP) in the Netherlands and the Tweede Algemene Waterpassing (TAW) in Belgium. The network of BMs realizes a vertical reference frame, which is only accessible at the BMs.

From a user point of view, the main disadvantage of spirit levelling is that it is time-consuming and expensive. From the government point of view, maintaining a network of BMs is labour-intensive and expensive; the heights of the BMs may change due to vertical land movement and BMs may be damaged or disappear. Both require regular surveys.

Therefore, governmental agencies in charge of providing vertical reference and users are interested in alternatives for vertical reference and height determination, respectively. Global Navigation Satellite Systems (GNSS) are widely seen as an alternative to spirit levelling, providing accuracies in line with the needs of the majority of users. Pre-requisite is that GNSS ellipsoidal heights can be transformed into national heights. Today's common practice to achieve this is to provide a (quasi-) geoid model in combination with a corrector surface. As the (quasi-) geoid model is not an interpolator to the vertical reference surface at the BMs, a corrector surface is computed, to account for systematic differences between the (quasi-) geoid model and the zero reference level at the BMs. The corrected (quasi-

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) geoid model is then used to transform GNSS ellipsoidal heights into national heights. The use of GNSS for height determination is currently the primary driver for improving the accuracy of (quasi-) geoid models.

A precise (quasi-) geoid model may also offer an alternative to a network of benchmarks as the realisation of a vertical reference frame. Recently, Canada has decided to use a gravimetric geoid model as the vertical datum (Véronneau and Huang, 2016). Then, GNSS ellipsoidal heights can be transformed directly into heights above the geoid without the need for a corrector surface.

In the framework of the project "Vertical Reference Frame for the Netherlands Mainland, Wadden Islands and Continental Shelf" (NEVREF), TU Delft computes a new quasi-geoid model for the Netherlands and Belgium. For the time being, the main motivation is to support levelling with GNSS. On the long term, it may also pave the way to a new vertical reference frame in these countries.

The heights of the BMs in the Netherlands and Belgium are levelled heights without gravity correction, and are referred to as NAP heights (in the Netherlands) and TAW heights (in Belgium), respectively. Inconsistencies caused by non-vanishing horizontal gravity gradients are below the noise level of spirit levelling in the Netherlands. In the hilly areas of Belgium, the inconsistencies are larger and may exceed the noise level in levelled height differences introducing some systematic distortions in the heights of the BMs. However, these distortions are much smaller than other systematic errors in the levelling networks, which will be discussed in Section 4.

In this paper, we quantify systematic errors in terrestrial gravity anomalies and airborne gravity disturbances, and investigate their impact on the quasi-geoid model for the Netherlands and Belgium. The paper is organised as follows: in Section 2, we discuss various sources of systematic errors in gravity and heights, and provide a simple formula based on Stokes' integral to obtain an order of magnitude estimate of their impact on the height anomalies. In Section 3 and 4, we quantify biases in the terrestrial and airborne gravity datasets and systematic errors in the heights of the gravity stations, respectively. Their influence on the quasi-geoid model for the Netherlands and Belgium is addressed in Section 5.

2 Impact of systematic errors in gravity and heights on height anomalies

Heck (1990) provides an extensive discussion of various error sources in gravity anomaly datasets. According to Heck (1990), the most critical errors are caused by inconsistencies in gravity datum, vertical datum, height systems, and horizontal datum. They may easily introduce systematic errors in the gravity anomalies, which may be nearly constant over larger areas.

The gravity anomaly datasets used in the computation of the quasi-geoid for the Netherlands and Belgium are from many different providers. Unfortunately, the metadata is not always complete. Some datasets comprise surface gravity values, but information about whether an atmospheric correction or a correction for permanent tides has been applied is frequently missing. Shipboard gravity datasets are known to be prone to systematic errors. Though we applied a cross-over adjustment and outlier detection to all shipboard datasets, residual systematic errors may still be present. Last but not least, the majority of gravity data are in the IGSN71, for some older gravity datasets this may not be the case.

Geopotential numbers are never provided. Instead, heights of the gravity stations are part of the datasets. Heights may refer to another epoch than the gravity measurements, meaning that vertical land movement between gravity data acquisition and levelling may introduce systematic errors when computing the normal gravity. For datasets from outside the Netherlands and Belgium, the relation between the corresponding vertical datum to the datum used in the Netherlands and Belgium, respectively, is not always exactly known, despite the efforts to unify height systems in the European Union (e.g., EVRF 2007). Some datasets are provided in terms of free-air gravity anomalies at the geoid. However, information about the computation of normal gravity (e.g., the normal gravity field used in the computation, or the heights used to reduce surface gravity to the geoid) is sometimes missing. Moreover, when computing a quasi-geoid, the free-air gravity anomalies need to be transformed into surface gravity anomalies, which strictly spoken requires information about the normal height. For some datasets, normal heights are not precisely known, which may introduce systematic errors in the surface gravity anomalies depending on the topography.

This, together with other error sources as discussed in Heck (1990) may introduce systematic yet unknown biases in the gravity anomaly datasets.

The impact of systematic errors in gravity anomalies on the height anomalies can be roughly estimated using Stokes' integral. The surface gravity anomalies used in the computation of the quasi-geoid model for the Netherlands and Belgium are defined as

$$\Delta g = g_{\mathrm{P}} - \gamma_{\mathrm{Q}}, \tag{2.1}$$

where g_{P} is gravity at the surface point P and γ_{Q} is normal gravity at the telluroid point Q. The telluroid used in this study is defined by the relation

$$C_{\mathrm{P}} = U_0 - U_{\mathrm{Q}}, \tag{2.2}$$

where C_{P} is the geopotential number of the surface point P, U_0 is the normal gravity potential at the surface of the GRS80 ellipsoid, and U_{Q} is the normal gravity potential at the telluroid point Q. As NAP heights and TAW heights are levelled heights, we may write $h_{\mathrm{Q}} = H_{\mathrm{P}} + \varepsilon$, where ε is the error caused by the use of levelled heights instead of normal heights. The ellipsoidal height of the telluroid point Q is related to the geopotential number at P as

$$h_{\mathrm{Q}} = \frac{C_{\mathrm{P}}}{\bar{\gamma}_{\mathrm{Q}}}, \tag{2.3}$$

with $\bar{\gamma}_{\mathrm{Q}}$ the mean value of normal gravity between the GRS80 ellipsoid and the telluroid point Q measured along the ellipsoidal normal through the associated surface point P. If the error in the levelled height H of a gravity station P is ε_{H}, the error in the surface gravity anomaly is

$$\varepsilon_{\Delta_{\mathrm{g}}} = -\frac{\partial \gamma}{\partial h}\Big|_0 \Big(\varepsilon + \varepsilon_{\mathrm{H}}\Big). \tag{2.4}$$

If a bias ε_{b} is present in the gravity anomaly, Eq. (2.4) is written as

$$\begin{aligned} \varepsilon_{\Delta_{\mathrm{g}}} &= \varepsilon_{\mathrm{b}} - \frac{\partial \gamma}{\partial h}\Big|_0 \Big(\varepsilon + \varepsilon_{\mathrm{H}}\Big) & (2.5) \\ &= \varepsilon_{\mathrm{b}} + 3.086 \cdot 10^{-6}\Big(\varepsilon + \varepsilon_{\mathrm{H}}\Big). & (2.6) \end{aligned}$$

The maximum impact of a systematic error $\varepsilon_{\Delta_{\mathrm{g}}}$ in a gravity anomaly dataset covering an area of size $s \times s$ km^2 on a height anomaly can be estimated using Stokes' integral:

$$\varepsilon_{\zeta_{[\mathrm{cm}]}} \approx 5.751 \cdot 10^{-2}\, s_{[\mathrm{km}]}\, \varepsilon_{\Delta_{\mathrm{g}[\mathrm{mGal}]}}. \tag{2.7}$$

Assuming that the gravity anomaly dataset has a systematic bias of ε_b and the heights of the gravity stations have a systematic bias of ε_{h}, we can use Eq. (2.6), and find for the maximum error in a height anomaly

$$\varepsilon_{\zeta_{[\mathrm{cm}]}} \approx s_{[\mathrm{km}]}\Big(5.751 \cdot 10^{-2}\, \varepsilon_{\mathrm{b}[\mathrm{mGal}]} + 1.775 \cdot 10^{-4}\, \varepsilon_{\mathrm{h}[\mathrm{cm}]}\Big). \tag{2.8}$$

For instance, assuming that a particular gravity anomaly dataset covers an area of 100×100 km^2, a bias of only $\varepsilon_{\mathrm{b}} = 0.1$ mGal causes already a maximum height anomaly error of 0.6 cm. A bias of $\varepsilon_{\mathrm{H}} = 1$ cm in the heights of this particular gravity anomaly dataset causes a maximum height anomaly error of just 0.02 cm. From this simple experiment, we can expect that biases in gravity datasets are critical in quasi-geoid modelling, whereas biases in the height network have a minor impact. Moreover, we may expect that inconsistencies in the heights of the gravity stations due to the use of levelled heights without gravity correction are negligible for quasi-geoid modelling in the Netherlands and Belgium. Therefore, we do not consider this source of error in this study, i.e., we assume that $\varepsilon = 0$ in Eqs. (2.4) and (2.6).

3 Systematic errors in gravity

As shown in Heck (1990), there are many contributors to systematic errors in gravity datasets. For the datasets used in the computation of the quasi-geoid model for the Netherlands and Belgium, it is not possible to identify and quantify the different contributors. Therefore, we decided to estimate per gravity dataset a bias parameter. This is straightforward when using weighted least-squares techniques in combination with a parametric model of the disturbing potential (e.g., a spherical radial basis function model). Details about the functional and stochastic model and the parameter estimation are provided in Farahani et al. (2017).

A total of 60 bias parameters were estimated. The identification number of the individual datasets, and a graphical rendition of the estimated bias parameters are shown in Figure 3.1. No bias parameter was esti-

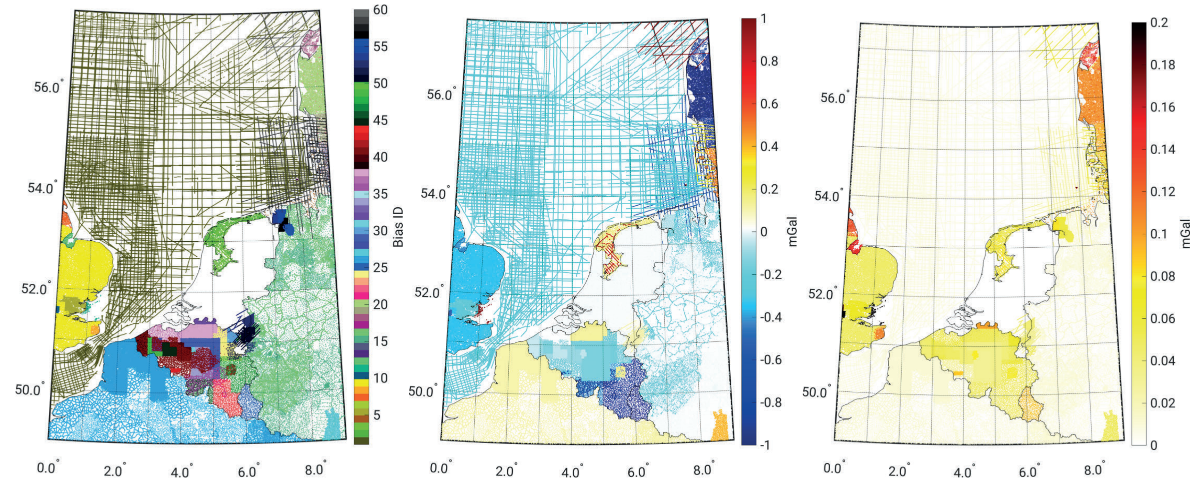

Figure 3.1: Left panel: Identification number of gravity dataset for which bias parameters were estimated. No bias parameter is estimated for the gravity dataset of the Dutch mainland. Middle panel: estimated bias parameters. Right panel: Standard deviations of the estimated bias parameters.

mated for the gravity dataset of the Dutch mainland. Therefore, the estimated bias parameters do not represent absolute biases, but biases relative to this dataset. Among the datasets are numerous shipboard datasets of different providers. As they were first crossover-adjusted and isolated tracks were removed, a single bias parameter was estimated for all shipboard gravity data. Bias parameters were also estimated for each individual airborne gravity dataset.

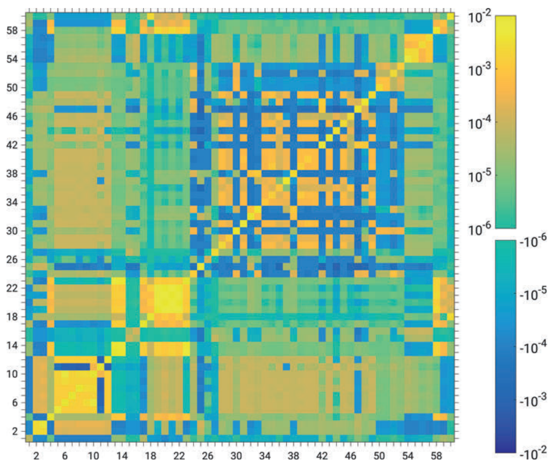

Figure 3.2: Noise covariance matrix of the estimated bias parameters.

Figure 3.2 shows the noise covariance matrix of the 60 estimated bias parameters. It reveals that the majority of bias parameters show little to moderate correlations. One exception are the highly correlated bias parameters no 19-22 (cf. Fig 3.1). Though this may give reason to estimate a single bias parameters for the four involved datasets, this has not been done in the results to be presented in Section 5, because the estimated bias parameters have similar amplitudes.

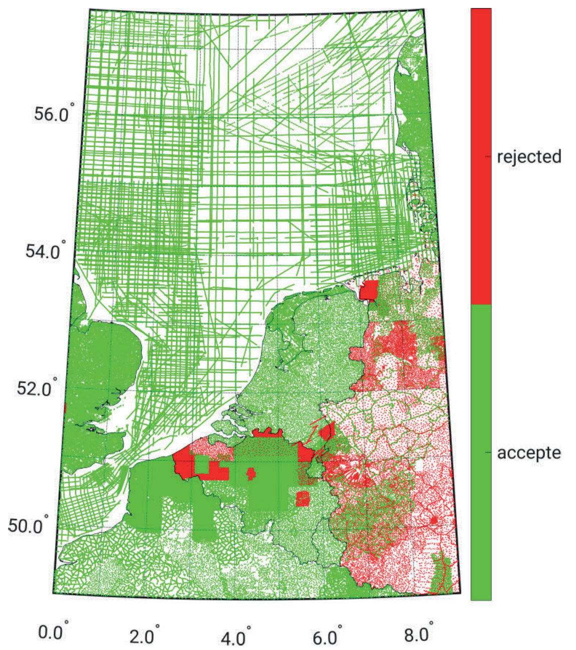

Figure 3.3: Spatial rendition of accepted and rejected (95% confidence level) bias parameters for the gravity datasets used in the computation of the quasi-geoid model for the Netherlands and Belgium.

The bias parameters were tested for statistical significance. At a 95% confidence level, 42 bias parameters turned out to be statistically significant; the remaining 18 bias parameters were rejected (cf. Fig 3.3 for a spatial rendition of accepted and rejected bias parameters). Figure 3.4 shows a histogram for the accepted and rejected bias parameters, respectively. The bias parameters range from -2.0 mGal to 1.5 mGal. Some bias parameters are striking such as the large bias for

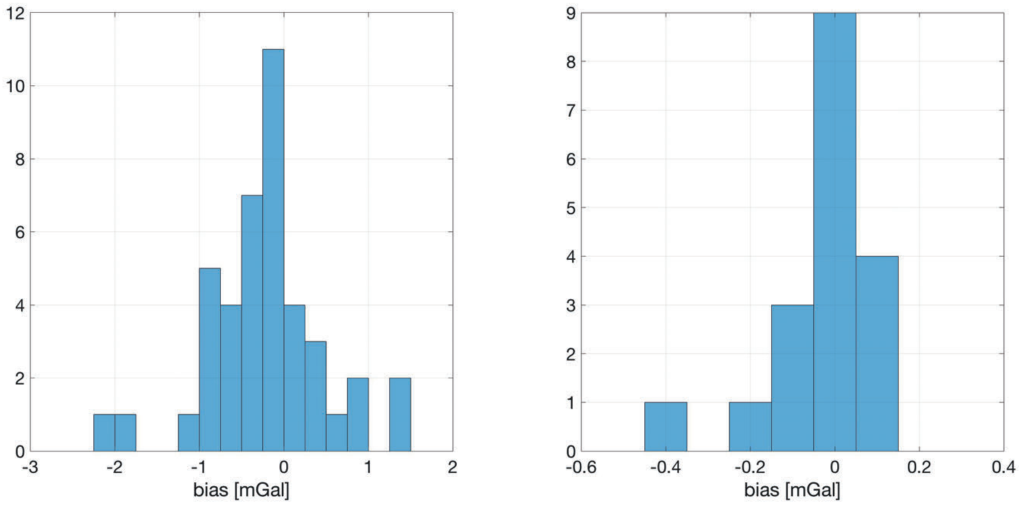

Figure 3.4: Histogram of the 42 statistically significant bias parameters (left panel) and the rejected bias parameters (right panel).

the Luxembourg gravity dataset of about −2.0 mGal and for the Danish gravity dataset of −1.2 mGal. For the Luxembourg gravity dataset, we lack any information concerning the vertical datum which the heights in the dataset refer to. As the data were originally acquired by the Observatoire Royal de Belgique and provided to us by the Belgian National Geographical Institute in Brussels, a possible explanation is that they refer to TAW rather than NG95 (the national height system in Luxembourg) as we assumed here. The difference is about the difference between NAP and TAW, i.e., 2.34 m, which would explain 35% of the estimated bias. A missing atmospheric correction could explain an additional 43% of the bias. The latter may also explain the largest share of the bias in the Danish gravity dataset.

The Belgian datasets have biases ranging from −1.75 to 0.12 mGal of unknown origin. The bias in the shipboard gravity anomaly dataset over the North Sea is −0.2 mGal; as the area is pretty large, the bias is expected to have a significant influence on the quasi-geoid model for the Netherlands and Belgium. The two shipboard gravity datasets over the IJssel lake/Wadden Sea have a bias of 0.15 mGal and 0.86 mGal, respectively. The latter number suggests that the atmospheric correction we applied was already applied by the data provider. Relatively large biases were found for the three airborne gravity datasets; 0.29, −0.55, and 1.41 mGal (identification numbers 58–60). The first two data sets were provided by the Bundesamt für Kartographie und Geodäsie (BKG) in Frankfurt; they are described in Schäfer et al. (2008). Based on a comparison with surface gravity data, Schäfer et al. (2008) report biases of 1.2 mGal and 1.3 mGal for the NorthGRACE-08 and BalGRACE-06 campaigns, respectively. Given these numbers, our biases are somewhat low, which may be explained by the fact that we used only a part of the two datasets. The other airborne gravity data set is the one obtained during the Skaggerak survey in 1996 mentioned in Olesen (2003). Unfortunately, no validation results regarding this dataset are available to the authors.

4 Systematic errors in heights

A part of the gravity data pre-processing consists in a transformation of the heights of all gravity stations to a single height datum. In our study, we adopted the EVRF2007. During the NEVREF project, several errors were found in the telluroid heights of the Dutch and Belgian gravity data points. We discovered that the NAP heights of the gravity stations were not adjusted for the deformations of the NAP height network as found in the fifth precise levelling campaign (Brand et al., 2004). This causes errors ranging from −3.0 to 1.7 cm (cf. left panel of Fig 4.1 for a spatial rendition of the errors). Recently, the Belgian National

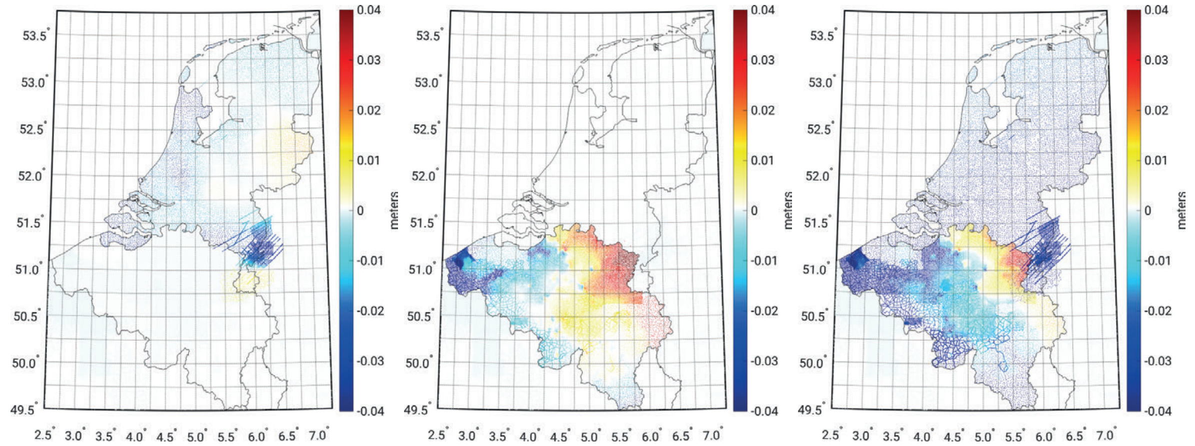

Figure 4.1: Left panel: Errors in the telluroid heights of the Dutch gravity data points introduced due to uncorrected deformations found in the fifth precise levelling campaign. Middle panel: Errors in the telluroid heights of the Belgian gravity data points, found recently after a re-adjustment of the entire TAW levelling network. Right panel: telluroid height changes when using the official EVRF2007 transformation parameters.

Geographical Institute did a re-adjustment of the entire TAW levelling network and found errors ranging from -12.0 to 5.0 cm (cf. middle panel of Fig 4.1). Finally, we found that the geopotential numbers of the Dutch first order levelling network, which were used in the computation of EVRF2007, were not correct. They turned out to be erroneous due to a wrong computation of the geopotential differences between the height markers, errors in the communication of the adopted tidal system in which the geopotential differences are expressed, and a mis-communication regarding the NAP datum point (Speth, 2016). One implication of the afore-mentioned errors is that the transformation parameters from TAW to EVRF2007 and NAP to EVRF2007, respectively, are not correct. For both countries, these transformation parameters account for the conversion of the mean permanent tide system (i.e., mean crust over mean geoid) adopted in NAP and TAW to the zero permanent tide system adopted in the EVRS as well as the datum shift between NAP/TAW and EVRF2007. In the final preprocessing scheme, we ignored the datum shift between NAP and EVRF2007, and only accounted for the difference in permanent tide. For the TAW heights, we applied the same transformation for permanent tide after transforming the TAW heights to NAP heights by subtracting 2.34 m. The right panel of Fig 4.1, shows a spatial rendition of the differences between the telluroid heights obtained in this way and the ones obtained when using the official transformation parameters of the EVRF2007. Note that for the Belgian data set, we applied the official transformation parameters to the unadjusted TAW heights.

5 Impact on the quasi-geoid model

The impact of two error sources on the quasi-geoid for the Netherlands and Belgium are analysed: i) systematic errors in the gravity datasets, which are modelled as a bias parameter per dataset (cf. Section 3), ii) errors in the height network of the Netherlands and Belgium including the effect they have on the transformation to EVRF2007 (cf. Section 4). The impact on the quasi-geoid model is defined as the difference with respect to a reference solution. The latter is the quasi-geoid model, which is computed using the statistically significant bias parameters and the latest version of the levelled heights for the gravity stations in the Netherlands and Belgium.

Figure 5.1 shows the impact of the estimated bias parameters for the gravity datasets used in the computation of the quasi-geoid model for the Netherlands and Belgium. As expected from the rough estimates of Section 2, the impact is very significant with a peak-to-peak amplitude of about 8 cm. The most prominent spatial pattern is a north-west south-east tilt in the quasi-geoid over the Belgian's mainland from 4 cm in the south-east to -1.5 cm along the coast. Over the Netherlands' mainland, the impact is much smaller, and ranges from 2 cm in the province of Limburg to -1.5 cm along large parts of the western coast. The largest impact is offshore near the coast of the province of Zeeland with a peak of about -4 cm. The impact on the quasi-geoid model of the 18 statistically not significant bias parameters, turned out to be below 2 mm.

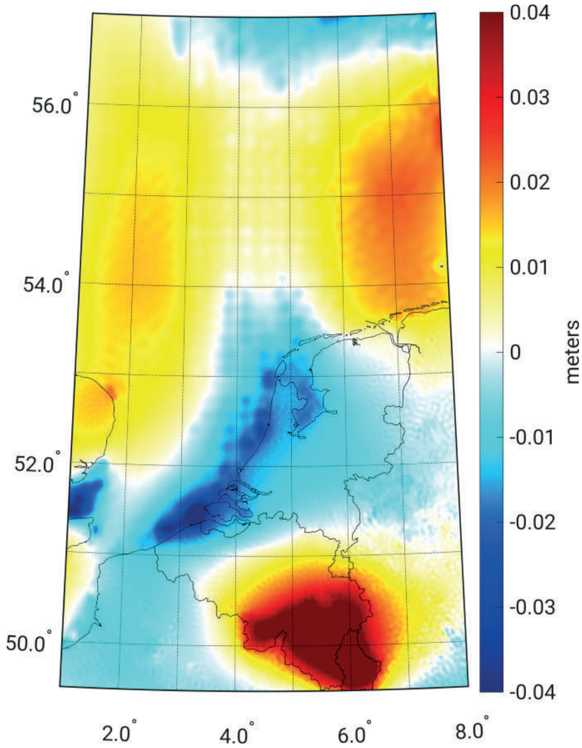

Figure 5.1: Impact of the estimated bias parameters in gravity datasets on the quasi-geoid model.

To investigate whether adding bias parameters to the functional model improves the quality of the estimated quasi-geoid model, we compared gravimetric height anomalies with geometric height anomalies at independent GPS/levelling points over the mainland of the Netherlands and Belgium. The statistics of the differences are shown in Table 5.1. For Belgium, the statistics improved significantly. For instance, the standard deviation (SD) of the differences reduced from 2.60 cm to 1.53 cm. For the Netherlands, the primary impact is on the mean, which increased from 1.38 cm to 1.95 cm. The standard deviation remained essentially unchanged.

Table 5.1: Statistics of differences between geometric and gravimetric height anomalies at GPS/levelling points. Per control dataset: first row: best quasi-geoid model; second row: quasi-geoid model without correcting for biases in gravity datasets; third row: quasi-geoid model based on bias-corrected gravity datasets. The control datasets comprise 3780 (Belgium) and 84 (NL) points, respectively.

	min [cm]	max [cm]	mean [cm]	RMS [cm]	SD [cm]
Belgium	−3.00	10.59	4.04	4.31	1.52
	−3.31	12.80	5.84	6.40	2.60
	−2.99	10.66	4.07	4.34	1.53
NL	−2.62	4.41	1.76	1.95	0.86
	−4.17	3.99	0.90	1.38	1.06
	−2.62	4.43	1.75	1.95	1.10

Figure 5.2 shows the impact on the computed quasi-geoid model of the errors in the telluroid heights of the Dutch and Belgian gravity data points, which are caused by the errors in the NAP and TAW

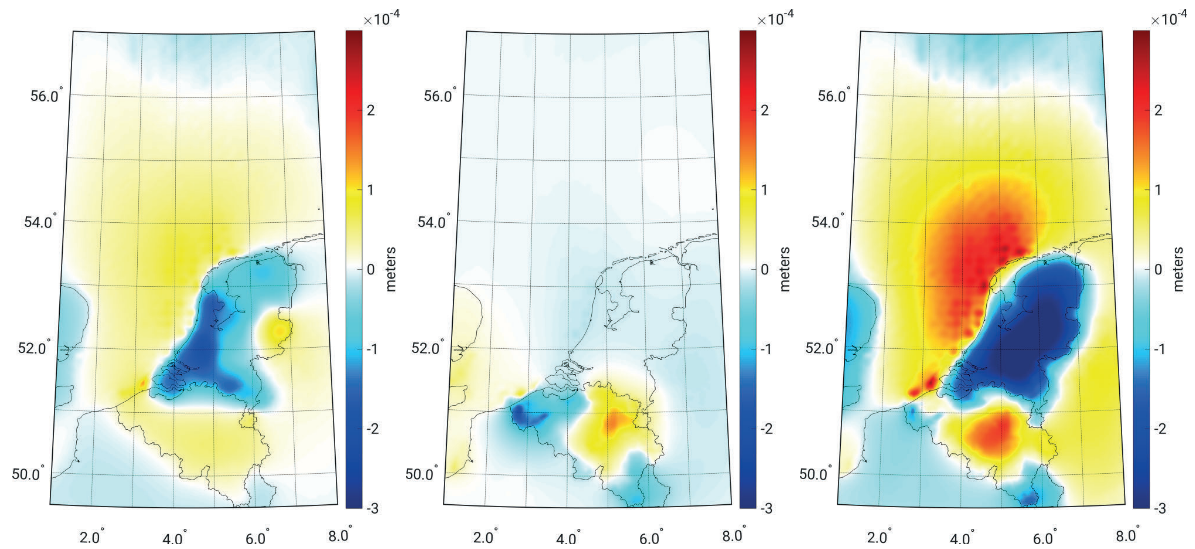

Figure 5.2: Impact of the height errors shown in Fig 4.1 on the quasi-geoid model. From left to right: i) errors in the telluroid heights of the Dutch gravity stations, ii) errors in the telluroid heights of the Belgian gravity stations, and iii) errors in the telluroid heights of the Dutch and Belgian gravity stations due to wrong EVRF2007 transformation parameters.

height networks and the transformation parameters to EVRF2007 (cf. Section 4). The impact of these errors is always below 0.3 mm. This is consistent with the order of magnitude estimate of Section 2. This also explains why the statistics of the differences of Table 5.1 are very similar for the best quasi-geoid model compared to the solution with bias-corrected gravity datasets.

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