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Anomalous Lorentz and CPT violation

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Abstract. If there exists Lorentz and CPT violation in nature, then it is crucial to discover and understand the underlying mechanism. In this contribution, we discuss one such mechanism which relies on four-dimensional chiral gauge theories defined over a spacetime manifold with topology $\mathbb{R}^3 \times S^1$ and periodic spin structure for the compact dimension. It can be shown that the effective gauge-field action contains a local Chern-Simons-like term which violates Lorentz and CPT invariance. For arbitrary Abelian $U(1)$ gauge fields with trivial holonomies in the compact direction, this anomalous Lorentz and CPT violation has recently been established perturbatively with a Pauli–Villars-type regularization and nonperturbatively with a lattice regularization based on Ginsparg-Wilson fermions.

1. Introduction

Experiment has shown the violation of P, C, CP, and T, but *not* of CPT. Indeed, there is the well-known CPT "theorem" (Lüders, 1954–57; Pauli, 1955; Bell, 1955; Jost, 1957), which states that any local relativistic quantum field theory is invariant under the combined operation (in whichever order) of charge conjugation (C), parity reflection (P), and time reversal (T). The formulation of the theorem can be sharpened, but the form stated suffices for the moment.

The main inputs of the theorem are

- flat Minkowski spacetime $(M, g_{\mu\nu}) = \left(\mathbb{R}^4, \eta_{\mu\nu}^{\text{Minkowski}} \right)$;
- invariance under proper orthochronous Lorentz transformations and spacetime translations;
- standard spin-statistics connection;
- locality and Hermiticity of the Hamiltonian.

We refer to two monographs [1, 2] for further details and references on the CPT theorem.

The following question arises: *can* CPT invariance be violated at all in a physical theory and, if so, *is* it in the real world? It was widely believed that only quantum-gravity or superstring effects could give CPT violation. But a different result has been obtained several years ago: for certain spacetime topologies and classes of chiral gauge theories, CPT invariance is broken anomalously, that is, by quantum effects.

The original paper for this "CPT anomaly" is Ref. [3]. Follow-up papers have appeared in Refs. [4, 5, 6, 7, 8] and an extensive review is given by Ref. [9]. The crucial ingredients of the CPT anomaly are

- chiral fermions and gauge interactions;
- non-simply-connected spacetime topology,
- periodic boundary conditions.

Expanding on the last two bullets, the spacetime manifold must have a separable compact spatial dimension (coordinate x^3) with periodic spin structure of the fermions. The main focus, up till now, has been on the topology $\mathbb{R}^3 \times S^1$, but also other topologies have been considered, for example topologies related to punctures or wormholes [7].

There are, at least, three possible applications of the CPT anomaly. First, there is the resulting optical activity of the vacuum, which may lead to observable effects for the Cosmic Microwave Background [9, 10, 11]. Second, there is the resulting fundamental arrow-of-time, which may play a role in explaining the "start" of the Big Bang [12]. Third, the CPT anomaly may also be used as a diagnostic tool for a hypothetical spacetime foam [7, 13].

In this short contribution, we focus on the basic mechanism of the CPT anomaly and skip possible applications (referring to the review [9] for further discussion of the phenomenology).

2. Heuristics

The CPT anomaly of a chiral gauge theory defined over the four-dimensional manifold $M = \mathbb{R}^3 \times S^1$, with trivial vierbeins $e^a_\mu(x) = \delta^a_\mu$ and appropriate background gauge fields, arises in four steps:

- a compact spatial dimension with coordinate $x^3 \in [0, L]$ and a periodic spin structure make that a chiral fermion can have a momentum component $p_3 = 0$;
- a single chiral 2-component Weyl fermion in four dimensions (4D) with $p_3 = 0$ corresponds to a single massless 2-component Dirac fermion in three dimensions (3D);
- a single massless Dirac fermion in 3D is known to have a "parity anomaly," provided gauge invariance is maintained exactly [14, 15, 16];
- this "parity" violation in 3D corresponds to T violation in 4D, which, in turn, implies CPT violation in 4D.

The actual CPT anomaly will be established by calculating the effective gauge-field action $\Gamma[A]$, where the effects of the virtual fermions have been integrated out, and by showing that this effective action $\Gamma[A]$ changes under a CPT transformation of the background gauge field A.

3. Perturbative calculation

3.1. Main result

Consider, for definiteness, a chiral gauge theory with the following gauge group G, left-handed-fermion representation R_L, and spacetime manifold M:

$$G = SO(10), \tag{1a}$$

$$R_L = N_{\text{fam}} \times (\mathbf{16}), \quad N_{\text{fam}} = 1, \tag{1b}$$

$$M = \mathbb{R}^3 \times S^1_{\text{PSS}}, \quad e^a_\mu(x) = \delta^a_\mu, \quad g_{\mu\nu}(x) = e^a_\mu(x)\, e^b_\nu(x)\, \eta_{ab} = \eta_{\mu\nu}, \tag{1c}$$

where the subscript "PSS" in (1c) stands for periodic spin structure and the metric $g_{\mu\nu}(x)$ has a Lorentzian signature $(-1, 1, 1, 1)$. The spacetime manifold M from (1c) is described by the following coordinates x^μ:

$$x^0, x^1, x^2 \in \mathbb{R}, \quad x^3 \in [0, L]. \tag{2}$$

The spinorial $SO(10)$ representation **16** from (1b) is a complex representation (see also the remarks in the first paragraph of Sec. 4.1). For simplicity, we have set N_{fam} in (1b) equal to 1, but similar results hold for the value $N_{\text{fam}} = 3$ relevant to elementary particle physics.

The fermionic field in the representation (1b) is denoted by $\psi_L(x)$ and the Lie-algebra-valued gauge field $A_\mu(x)$ is defined as follows:

$$A_\mu(x) \;\equiv\; g\, A_\mu^a(x)\, T^a\,, \tag{3a}$$

$$\mathrm{tr}\left(T^a\, T^b\right) \;=\; -\frac{1}{2}\, \delta^{ab}\,, \tag{3b}$$

with argument x standing for $(x^0,\, x^1,\, x^2,\, x^3)$, Yang–Mills coupling constant g, and real gauge fields $A_\mu^a(x)$. The index "a" in (3a) labels the anti-Hermitian generators T^a of the so(10) Lie algebra and is summed over ($a = 1,\,\ldots,45$), while (3b) specifies the normalization of the generators.

Both gauge and fermionic fields are assumed to have periodic boundary conditions in x^3,

$$A_\mu(\widetilde{x},\, x^3 + L) \;=\; A_\mu(\widetilde{x},\, x^3)\,, \tag{4a}$$

$$\psi_L(\widetilde{x},\, x^3 + L) \;=\; \psi_L(\widetilde{x},\, x^3)\,, \tag{4b}$$

with $\widetilde{x} \equiv (x^0,\, x^1,\, x^2)$. The coupling between gauge and fermionic fields is specified by the following Dirac equation (equivalent to the standard Weyl equation [17] by use of the chiral representation of the Dirac matrices γ^μ):

$$\gamma^\mu \left(\partial_\mu + A_\mu(x)\right) \psi_L(x) = 0\,, \tag{5}$$

where the coupling constant g has been absorbed in the gauge field according to (3a).

For this setup, the complete effective gauge-field action $\Gamma[A]$ for $A \in$ so(10) is, of course, not known exactly (there are certain exact results in 2D [4, 5]). But the crucial term in $\Gamma[A]$ has been identified perturbatively [3, 8] for a gauge field with trivial holonomies (e.g., $A_3 = 0$):

$$\Gamma_{\mathrm{anom}}^{\mathbb{R}^3 \times S^1}[A] \;=\; \frac{1}{16\,\pi^2} \int_{\mathbb{R}^3} dx^0 dx^1 dx^2 \int_0^L dx^3\, \frac{\pi}{L}\, \epsilon^{\kappa\lambda\mu 3}\, \mathrm{tr}\left(A_\kappa(x)\, A_{\lambda\mu}(x) - \frac{2}{3}\, A_\kappa(x)\, A_\lambda(x)\, A_\mu(x)\right)$$
$$+ \ldots\,, \tag{6}$$

in terms of the Levi-Civita symbol $\epsilon^{\kappa\lambda\mu\nu}$, the Lie-algebra-valued gauge field $A_\kappa(x)$ from (3a), and the corresponding Yang–Mills field strength $A_{\lambda\mu}(x)$,

$$A_{\lambda\mu}(x) \;\equiv\; \partial_\lambda A_\mu(x) - \partial_\mu A_\lambda(x) + A_\lambda(x)\, A_\mu(x) - A_\mu(x)\, A_\lambda(x)\,. \tag{7}$$

The ellipsis in (6) contains further (nonlocal) terms; see the third remark in Sec. 3.2.

The result (6) has been obtained perturbatively at the one-loop level for arbitrary gauge fields with trivial holonomies,

$$\forall \widetilde{x} \in \mathbb{R}^3 \,:\, H_3(\widetilde{x}) = \mathbb{1}\,, \tag{8}$$

where the holonomy along the straight path in the 3 direction starting at $\widetilde{x} \equiv (x^0,\, x^1,\, x^2)$ and $x^3 = 0$ is defined by

$$H_3(\widetilde{x}) \equiv \mathcal{P} \exp\left[\int_0^L dy\, A_3(\widetilde{x},\, y)\right] \in G\,, \tag{9}$$

with the path-ordering operator $\mathcal{P}$. Taking into account the periodic boundary conditions (4a), the trace of the holonomy $H_3(\widetilde{x})$ corresponds to a gauge-invariant quantity, also known as the Wilson loop variable.

The spacetime integral in (6) involves the Chern-Simons density [18],

$$\omega_{\mathrm{CS}}[A_0,\, A_1,\, A_2] \equiv \frac{1}{16\,\pi^2}\, \epsilon^{klm}\, \mathrm{tr}\left(A_k\, A_{lm} - \frac{2}{3}\, A_k\, A_l\, A_m\right)\,, \tag{10}$$

where the repeated indices k, l, and m are summed over $\{0, 1, 2\}$. A genuine topological Chern–Simons term $\Omega_{\mathrm{CS}} = \int \omega_{\mathrm{CS}}$ is obtained only for a 3-dimensional spacetime manifold [18]. Our anomalous action term (6) holds, however, in four spacetime dimensions: the integration is over four spacetime coordinates and the gauge fields A_μ also have a dependence on x^3. Hence, the qualification "Chern–Simons-like" for the anomalous local action term in (6). This local action term is nontopological in the sense that there is a nontrivial dependence on the spacetime metric or vierbein; see, e.g., Ref. [19] and also the further remarks in the last paragraph of Sec. 5.

The local term (6) is *Lorentz-noninvariant*, because of the explicit spacetime index "3" entering the Levi–Civita symbol, and is also *CPT-odd*, because of the odd number (namely, three) of spacetime indices for the gauge-field terms in the large brackets. Recall that the standard Yang–Mills action density term $\mathrm{tr}\left(A_{\mu\nu}(x) A^{\mu\nu}(x)\right)$ is Lorentz-invariant and CPT-even [17].

3.2. Technical remarks

In this subsection, we present six technical remarks, which can, however, be skipped in a first reading. First, the non-Abelian result (6) also holds for an Abelian $U(1)$ group, as long as attention is given to the proper normalization. Indeed, the single Lie algebra generator T^1 can be written as $T^1 = \left(i/\sqrt{2n}\right) \mathbb{1}_n$, in terms of an $n \times n$ identity matrix $\mathbb{1}_n$. Now take $n = 1$, so that the trace operation in (6) becomes trivial. The covariant derivative (5) on a single left-handed fermion χ_L then reads $\left(\partial_\mu + i\left(g/\sqrt{2}\right) A_\mu^1\right) \chi_L$ and we can define the Abelian $U(1)$ gauge field $B_\mu(x) \equiv A_\mu^1(x)$ and the $U(1)$ charge $e \equiv g/\sqrt{2}$. This rather explicit discussion aims to clarify some confusing statements in Ref. [3], where the parameter "a" in Eqs. (3.13) and (4.1) is simply to be omitted.

Second, the following expression is obtained by writing $\epsilon^{\kappa\lambda\mu 3}$ in the integrand of (6) as $\epsilon^{\kappa\lambda\mu\nu} \partial_\nu x^3$ and performing a partial integration (for gauge fields vanishing at infinity):

$$\Gamma_{\mathrm{anom}}^{\mathbb{R}^3 \times S^1}[A] = \frac{1}{32\,\pi} \int_{\mathbb{R}^3} dx^0 dx^1 dx^2 \int_0^L dx^3 \, \frac{x^3}{L} \, \epsilon^{\kappa\lambda\mu\nu} \, \mathrm{tr}\left(A_{\kappa\lambda}(x) \, A_{\mu\nu}(x)\right) + \dots, \qquad (11)$$

where the integrand depends solely on the Yang–Mills field strength (7). The spacetime-dependent "coupling constant" x^3/L in (11) makes clear that Lorentz invariance is broken.

Third, let us briefly discuss the issue of non-Abelian gauge invariance (further details can be found in Sec. 4 of Ref. [3]). The integrand of the non-Abelian Chern–Simons-like term in (6) is not completely gauge invariant (invariant only under infinitesimal 4D gauge transformations) but is manifestly periodic in x^3 because of the boundary conditions (4a). The integrand of the non-Abelian term (11) is completely gauge invariant (invariant also under "large" 3D gauge transformations and infinitesimal 4D deformations thereof) but is not manifestly periodic in x^3 (in addition, the single coordinate x^3 is not a "good" coordinate and two patches are needed to cover the circle S^1). Incidentally, the nonlocal terms of the ellipsis in (6) are precisely there to restore the full non-Abelian gauge invariance (cf. Refs. [15, 16]).

Fourth, the anomalous Chern–Simons-like term (6) for non-Abelian gauge fields has, strictly speaking, only been derived [3] for a special class of background fields, namely x^3-independent gauge fields with vanishing component in the compact direction,

$$A_3 = 0, \qquad (12a)$$

$$A_{\widetilde{\mu}} = A_{\widetilde{\mu}}(\widetilde{x}), \qquad (12b)$$

with definition $\widetilde{x} \equiv (x^0, x^1, x^2)$ and $\widetilde{\mu}$ taking values from the set $\{0, 1, 2\}$.

Fifth, it is possible to consider a larger class of background fields, namely x^3-dependent

Abelian $U(1)$ gauge fields $B_\mu(x) \in \mathbb{R}$ with vanishing component in the compact direction,

$$B_3 = 0\,, \tag{13a}$$

$$B_{\widetilde{\mu}} = B_{\widetilde{\mu}}(\widetilde{x}, x^3)\,, \tag{13b}$$

$$B_{\widetilde{\mu}}(\widetilde{x}, x^3) = B_{\widetilde{\mu}}(\widetilde{x}, x^3 + L)\,, \tag{13c}$$

together with periodic boundary conditions (4b) on the relevant fermionic fields $\chi_L(x)$. For Abelian $U(1)$ gauge fields (13) with definitions from the first remark of this subsection and with a Pauli–Villars-type regularization [20], the Abelian version of the local Chern–Simons-like term in (6) has been obtained [8],

$$\Gamma^{\mathbb{R}^3 \times S^1}_{\text{anom}}[B] = \frac{1}{16\,\pi^2}\, F\, e^2 \int_{\mathbb{R}^3} dx^0 dx^1 dx^2 \int_0^L dx^3\, \frac{2\,\pi}{L}\, \epsilon^{\kappa\lambda\mu3}\, B_\kappa(x)\, \partial_\lambda B_\mu(x) + \dots\,. \tag{14}$$

The numerical factor F in (14) counts the sum of the squares of the fermion charge in units of e; see also Sec. 4.2. The derivation of (14) requires that the typical momenta of the background fields (13) are very much less than the regulator masses.

Sixth, the anomalous term (6) can be generalized by including an additional prefactor $(2\,k^{(0)} + 1)$ for $k^{(0)} \in \mathbb{Z}$, due to the freedom in defining the regularized theory. This freedom is particularly clear in the lattice formulation to be discussed in the next section, where the fermion measure of the path integral is characterized by an integer $k^{(0)}$.

4. Nonperturbative calculation

4.1. Main result

Consider, for definiteness, a four-dimensional Abelian chiral gauge theory with

$$G = U(1), \tag{15a}$$

$$R_L = 6 \times \left(\frac{1}{3}\right) + 3 \times \left(-\frac{4}{3}\right) + 3 \times \left(\frac{2}{3}\right) + 2 \times \left(-1\right) + 1 \times \left(2\right) + 1 \times \left(0\right)\,. \tag{15b}$$

The perturbative chiral gauge anomalies of this particular theory cancel out, because the group (15a) is a subgroup of the "safe" group $SO(10)$ and the reducible representation (15b) of $U(1)$ is contained in the irreducible representation **16** of $SO(10)$. Observe that R_L from (15b) is a complex representation: there are, for example, six left-handed fermions with a normalized charge $+1/3$ but none with a normalized charge $-1/3$. This may be compared to the vectorlike gauge theory of quantum electrodynamics with the real representation $R_L = (+1) + (-1)$.

Next, define a chiral lattice gauge theory over a finite hypercubic lattice with

- periodic spin structure in one direction;
- Ginsparg–Wilson fermions [21];
- Neuberger's lattice Dirac operator [22, 23];
- Lüscher's chiral constraints [24, 25].

The fermionic fields are, as usual, associated with the lattice sites and the gauge fields with the directed links between neighbouring lattice sites. Specifically, the fields are denoted by $\psi_L(x)$ at the lattice site x and $U_\mu(x) \in G$ for the link in the μ direction starting from the lattice site x.

The Euclidean effective gauge-field action $\Gamma[U]$ is given by the Euclidean path integral over the fermionic fields $\psi_L(x)$. The goal is to establish that this effective gauge-field action $\Gamma[U]$ changes under a CPT transformation,

$$\Gamma[U] \neq \Gamma[U^{\text{CPT}}]\,, \tag{16}$$

where U denotes the set of link variables of the lattice.

The result (16) has been established nonperturbatively [6, 8] for Abelian $U(1)$ gauge fields with trivial holonomies (e.g., $U_3 = 1$). Moreover, the origin of the CPT anomaly has been identified as an ambiguity [6] in the choice of basis vectors for the fermion integration measure; cf. the path-integral derivation of the triangle (Adler–Bell–Jackiw) anomaly [26].

The result (16) has, in particular, been obtained [8] for $U(1)$ link variables that are x^3-dependent and trivial in the compact direction,

$$U_3 = 1\,, \tag{17a}$$

$$U_{\widetilde{\mu}} = U_{\widetilde{\mu}}(\widetilde{x}, x^3)\,, \tag{17b}$$

$$U_{\widetilde{\mu}}(\widetilde{x}, x^3) = U_{\widetilde{\mu}}(\widetilde{x}, x^3 + L)\,, \tag{17c}$$

with definition $\widetilde{x} \equiv (x^0, x^1, x^2)$ and $\widetilde{\mu}$ taking values from the set $\{0, 1, 2\}$. All arguments in (17) are understood to coincide with the points of the four-dimensional hypercubic lattice.

4.2. Continuum limit
In addition to establishing the existence of the CPT anomaly nonperturbatively, we have also investigated the continuum limit, with lattice spacing $a \to 0$ and number of links in the 3 direction $N_3 \to \infty$, while keeping $L \equiv N_3\, a$ at a constant value. For slowly-varying link variables $U_\mu(x) \approx \exp[i\,e\,a\,B_\mu(x)]$ in the continuum limit, the anomalous term $\Gamma_{\mathrm{anom}}[U]$ from the nonperturbative lattice calculation has been shown [8] to give rise to the Abelian Chern–Simons-like term (14) with a numerical factor $F \equiv \sum_f (q_f/e)^2 = 40/3$ for the representation (15b). As mentioned in the last remark of Sec. 3.2, there is an additional prefactor $(2\,k^{(0)} + 1)$ coming from the definition of the path-integral fermion measure characterized by an integer $k^{(0)}$ (see App. B of Ref. [6]).

Hence, the nonperturbative lattice result does not modify the result of the perturbative one-loop calculation as discussed in Sec. 3. This is certainly reminiscent of the Adler–Bardeen result for the triangle anomaly [27], but it remains to be confirmed that there arise no additional terms in the nonperturbative lattice calculation.

5. Outlook
The subtle role of topology for the local properties of quantum field theory is well-known, the prime example being the Casimir effect. For certain chiral gauge theories, we now have established that the interplay of ultraviolet and infrared effects may also lead to Lorentz and CPT noninvariance, even for flat spacetime manifolds, that is, without local gravity.

Let us end with two general remarks. First, we have found an anomalous origin of Lorentz and CPT violation, which ultimately traces back to the regularization in the ultraviolet, operating at ultrahigh energies or ultra-small distances (see, e.g., the discussion in Sec. V of Ref. [8]). Regularization is typically used as a mere mathematical device, to be removed after the calculation. But it is also possible that a particular regularization has a deeper physical meaning. An example is the lattice regularization, which could be directly relevant if spacetime possesses a fundamental discreteness.

Second, the question arises how the anomalous photon term (14) affects gravity [28, 29]. The problem, now, is that the corresponding term in the energy-momentum tensor is, in general, neither symmetric nor conserved. There are solutions of the Einstein equation for special cases [29] but not for the general case. At this moment, it is not clear how the gravitational theory can be extended in order to allow for an anomalous Chern–Simons-like-term in the effective gauge-field action.

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