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Initial-Boundary Value Problem for Hyperbolic Equation with Singular Coefficient and Integral Condition of Second Kind

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Abstract—We research an initial-boundary value problem with integral condition of the second kind in a rectangular domain for a hyperbolic equation with singular coefficient. The solution is obtained in the form of the Fourier—Bessel series. There are proved theorems on uniqueness, existence and stability of the solution. In order to prove the existence of solution of the non-local problem we obtain sufficient conditions for the convergence of the series in terms of the initial values.

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1. INTRODUCTION

Let $l, T > 0$ be given real values, $D = \{(x,t)|\ 0 < x < l,\ \ 0 < t < T\}$ is rectangular domain. We consider hyperbolic equation

$$\Box_B u(x,t) \equiv u_{tt} - u_{xx} - \frac{k}{x} u_x = 0, \tag{1}$$

where $k \neq 0$ is given real number. Equation (1) belongs to the class of degenerated hyperbolic equations. Investigation of boundary value problems for that equations is of importance for contemporary theory of differential equations with partial derivatives. The problems have numerous applications in gas dynamics, magnet hydrodynamics, envelope theory and other fields of science and technique.

The Cauchy and Cauchy—Goursat problems for equation (1) were studied first in the work [1] for all $k \geq 1$ in characteristic triangle. As shown in the paper [2], the problems are not well-posed for $k < 0$. And the papers [3, 4] contain studies of the problems for equations of mixed type such that their hyperbolic parts coincide with equation (1). The non-local problems for equation (1) with integral conditions of the first kind and second kind are studied in the papers [5—7].

In the present paper we investigate the following initial-boundary value problem for equation (1) in domain D with non-local integral condition of the second kind for $k \leq -1$. We put in the further consideration without loss of generality $l = 1$, because equation (1) is invariant with regard to change of variables $x_1 = x/l$, $y_1 = y/l$.

Statement of the problem. It is necessary to find the function $u(x,t)$ satisfying the following restrictions:

$$u(x,t) \in C^1(\overline{D}) \cap C^2(D), \tag{2}$$

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