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New Size Hierarchies for Two Way Automata

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Abstract—We introduce a new type of nonuniform two-way automaton that can use a different transition function for each tape square. We also enhance this model by allowing to shuffle the given input at the beginning of the computation. Then we present some hierarchy and incomparability results on the number of states for the types of deterministic, nondeterministic, and bounded-error probabilistic models. For this purpose, we provide some lower bounds for all three models based on the numbers of subfunctions and we define two witness functions.

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1. INTRODUCTION

Nonuniform models (like circuits, branching programs, uniform models using advice, etc.) have played significant roles in computational complexity, and, naturally they have also been investigated in automata theory (e.g., [1–4]). The main computational resource for nonuniform automata is the number of internal states that depends on the input size. Thus we can define linear, polynomial, or exponential size automata models. In this way, for example, nonuniform models allow us to formulate the analog of "P versus NP problem" in automata theory: Sakoda and Sipser [5] conjectured that simulating a two–way nondeterministic automaton by two–way deterministic automata requires exponential number of states in the worst case. But, the best known separation is only quadratic ($O(n^2)$) [6, 7] and the researchers have succeeded to obtain slightly better bounds only for some modified models (e.g., [8–11]). Researchers also considered similar question for OBDD model that can be seen as nonuniform automata (e.g., [12–17]).

In this paper, we present some hierarchy results for deterministic, nondeterministic, and bounded-error probabilistic nonuniform two–way automata models, which can also be seen as a "two–way" version of ordered binary decision diagrams (OBDDs) [18]. For each input length (n), our models can have different number of states, and, like Branching programs or the data-independent models defined by Holzer [2], the transition functions can be changed during the computation. Holzer's model can use a different transition function for each step. We restrict this property so that the transition function is the same for the same tape positions, and so, we can have at most n different transition functions. Moreover, we enhance our models by shuffling the input symbols at the beginning of the computation. We give the definitions and related complexity measures in Section 2.

In order to obtain our main results, we start with presenting some generic lower bounds (Section 3) by using the techniques given in [19, 20]. Then, we define two witness Boolean functions in Section 3.3.2:

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