ISSN 1995-0802, Lobachevskii Journal of Mathematics, 2018, Vol. 39, No. 7, pp. 1016–1023. © Pleiades Publishing, Ltd., 2018.

On the Probability of Finding Marked Connected Components Using Quantum Walks

K. Khadiev[1,2*], N. Nahimovs[1], and R. A. M. Santos[1***]**

(Submitted by F. M. Ablayev)

[1] *Center for Quantum Computer Science, Faculty of Computing, University of Latvia, Raina bulv. 19, Riga, LV-1586, Latvia*

[2] *Institute of Computational Mathematics and Information Technologies, Kazan (Volga region) Federal University, ul. Kremlevskaya 35, Kazan, Tatarstan, 420008 Russia*

Received February 6, 2018

Abstract—Finding a marked vertex in a graph can be a complicated task when using quantum walks. Recent results show that for two or more adjacent marked vertices search by quantum walk with Grover's coin may have no speed-up over classical exhaustive search. In this paper, we analyze the probability of finding a marked vertex for a set of connected components of marked vertices. We prove two upper bounds on the probability of finding a marked vertex and sketch further research directions.

DOI: 10.1134/S1995080218070144

Keywords and phrases: *Quantum computing, quantum walks, exceptional configurations, stationary states.*

1. INTRODUCTION

Searching is an important problem in Computer Science. Using Grover's quantum algorithm [1] one can solve the unstructured search problem quadratically faster than classically. A quadratic speed-up is also obtained when searching for a single marked vertex in some classes of graphs by using quantum walks [2–4]. In case of multiple marked vertices the situation gets more tricky. Krovi et al. [4] gave a quantum walk based algorithm that achieves the quadratic speed-up for any reversible and ergodic Markov chain and showed that for multiple marked vertices it can search quadratically faster than a quantity called the "extended hitting time," which is equivalent to the hitting time for one marked vertex and lower-bounded by it. Recently, Hoyer and Komeili [5] described a quantum walk based algorithm for finding multiple marked vertices in the two-dimensional lattice. Their algorithm uses quadratically fewer steps than a random walk on the two-dimensional lattice, ignoring logarithmic factors. On the other hand, for some quantum walk based search algorithms additional marked vertices can make the search easier or harder depending on the placement of marked vertices [6].

In this paper, we consider search by coined discrete- time quantum walk [7] on general graphs with multiple marked vertices. Suppose we have a graph $G = (V, E)$ with a set of vertices V and a set of edges E. Let $n = |V|$ and $m = |E|$. The discrete-time quantum walk on G has associated Hilbert space $\mathcal{H}^{2m}$ with the set of basis states $\{|v, c\rangle : v \in V, 0 \le c < d_v\}$, where d_v is the degree of vertex v. The evolution operator is the product of the coin operator followed by the shift operator, that is, $U = S \cdot C$. The coin transformation C is the direct sum of coin transformations for individual vertices, i.e. $C = C_{d_1} \bigoplus \cdots \bigoplus C_{d_n}$ with C_{d_i} being the Grover diffusion transformation of dimension d_i. The shift

*E-mail: kamilhadi@gmail.com
**E-mail: nikolajs.nahimovs@lu.lv
***E-mail: rsantos@lu.lv