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Uniform Wavelet-Approximation of Singular Integral Equation Solutions

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Abstract—In this article we consider a singular integral equation of the first kind with a Cauchy kernel on a segment of the real axis, which is a mathematical model of many applied problems. It is known that such an equation is exactly solved only in rare cases, therefore, the problem of its approximate solution with obtaining uniform error estimates is very actual. This equation is considered on a pair of weighted spaces that are constrictions of the space of continuous functions. The correctness of the problem of solving this equation on a chosen pair of spaces of the desired elements and right-hand sides gives the possibility of its approximate solution with a theoretical justification. The numerical method proposed in this article is based on the approximation of the unknown function by Chebyshev wavelets of the second kind. Uniform error estimates are established depending on the structural properties of the initial data. The numerical experiment in the Wolfram Mathematica package showed a good convergence rate of the approximate solution to the exact one.

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1. INTRODUCTION

In many areas of science and technology there is a singular integral equation (s.i.e.) of the first kind with a Cauchy kernel of the following form

$$Kx \equiv \frac{1}{\pi}\int_{-1}^{1}\frac{\sqrt{1-\tau^2}x(\tau)}{\tau-t}d\tau + \frac{1}{\pi}\int_{-1}^{1}\sqrt{1-\tau^2}h(t,\tau)x(\tau)d\tau = f(t), \quad |t|<1, \tag{1}$$

where $h(t,\tau)$, $f(t)$ are known continuous functions in their domains of definition, $x(\tau)$ is the desired function, and the singular integral

$$I\varphi \equiv I(\varphi;t) = \frac{1}{\pi}\int_{-1}^{1}\frac{\varphi(\tau)}{\tau-t}d\tau$$

is understood in the sense of Cauchy's principal value.

From the theory of such equations it follows that to find the exact solution of a s.i.e. (1) in closed form is possible only in individual cases [1, 2]. Therefore, various approximate methods for solving it have been developed, which, as a rule, are based on polynomial approximation of the exact solution (see, for example, in [3, 4]). It should be noted that the main difficulty in solving singular integral equations of the first kind is related to the incorrectness of the problem of their solutions on many pairs of function spaces, including the space of continuous functions, which in its turn is connected with the unboundedness of

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