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The Problem of Projecting the Origin of Euclidean Space onto the Convex Polyhedron

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Abstract—This paper is aimed at presenting a systematic exposition of the existing now different formulations for the problem of projection of the origin of the Euclidean space onto the convex polyhedron (PPOCP). We have concentrated on the convex polyhedron given as a convex hull of finitely many vectors of the space. We investigated the reduction of the projection program to the problems of quadratic programming, maximin, linear complementarity, and nonnegative least squares. Such reduction justifies the opportunity of utilizing a much more broad spectrum of powerful tools of mathematical programming for solving the PPOCP. The paper's goal is to draw the attention of a wide range of research at the different formulations of the projection problem.

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1. INTRODUCTION

The problem of projecting the origin of the Euclidean space onto a convex polyhedron plays an invaluable role in the differentiable and nondifferentiable convex optimization, in the theory of linear separation the convex polyhedra, data classification and identification. For many years, the numerous applications of the PPOCP motivates intensive studying the possibility of reducing this problem (in the original formulation with an abstract constraint) to the other related ones of mathematical programming [1–4]. Such a reduction, as a rule, tends to encourage the appearance of new methods for solving the original problem and finding new applications for the already existing methods.

Indeed, the class of projection techniques is successfully used for solving a variety of the mathematical problems. The methods applying the concept of projection are commonly referred to as the projection methods [5]. These algorithms use different kinds of projections onto the convex sets (including orthogonal projections which are the subject of this article), and serve for solving the optimization problems as well as the so-called feasibility problems (the problems of finding the point belonging to some set). The adequate for the most applications of the projection methods up-to-date overview of the literature is presented in [5, 6]. In these sources, there is annotated the bibliography on the iterative projective techniques that use a projection onto the convex and closed sets specified by a finite family of sets (for example, the intersection of the individual sets from this family). The computational success of these methods of projection is determined by the fact that, as a rule, it is always easier to implement the projection onto the individual sets with a simpler structure than, for example, at their intersection.

Finding the projection of points of the space onto a convex set, including a convex polyhedron, is used in particular in the method of cyclic projections. The cyclic orthogonal projection method can serve, for example, to find a point belonging to the set $\Phi = \Phi_1 \cap \Phi_2 \cap \ldots \cap \Phi_N$, where Φ_i, $i = \overline{1, N}$ are the polyhedra specified by the convex hulls of the finitely many vectors [6, 7]. We further note that the problem of projection of some point $p \in \mathbb{R}^n$ onto the convex polyhedron L may be reduced to the problem

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