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On Hamburger Moments Problem Generated by a Group With Two Limit Points

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Abstract—We study a linear four-element equation in the class of solutions that are holomorphic outside an isosceles trapezium and vanish at infinity. The equation is used here to investigate the Hamburger moments problem for entire functions of exponential type.

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INTRODUCTION

It is well-known that the number of limit points of a properly discontinuous group of fractional-linear transformations can take four values 0, 1, 2, and ∞, only ([1], Chap. 14, § 7). The problem of Stiltjes moments and its generalization on the case of multiple rays for entire functions of exponential type (e. f. e. t.) in the two first cases were considered earlier in papers [2] and [3], respectively. The goal of the present paper is to investigate the case of a group with two limit points.

Let D be a isosceles trapezium with vertices $t_1 = 1 - i$, $t_2 = \lambda t_1$, $t_3 = \overline{t}_2$, $t_4 = \overline{t}_1$ and sides ℓ_j enumerated in the order of positive detour of boundary $\Gamma = \partial D$. Here $\lambda > 1$ and $t \in \ell_1 \Rightarrow \arg t = -\pi/4$. If from the closure $\overline{D}$ we delete a "half of boundary", then we obtain the fundamental set of a properly discontinuous group with generative transformations $\sigma_1(z) = iz$, $\sigma_2(z) = \lambda^{-1} z$. This group has two limit points 0 and ∞. On the boundary, the generating transformations and their inverse transformations induce the involutive shift $\alpha(t) = \{\sigma_j(t),\ t \in \ell_j\}$, where $\sigma_3 = \sigma_1^{-1}$ and $\sigma_4 = \sigma_2^{-1}$, which changes the orientation of Γ and has the discontinuity points of the first kind at the vertices.

In Item 1 we study the functional equation

$$(Vf) \equiv \sum_{j=1}^{4} (-1)^{j+1} f[\sigma_j(z)] = g(z), \quad z \in D, \tag{1}$$

under the following assumptions.

1) The free member $g(z)$ is holomorphic in D and its boundary values $g^+(t)$ satisfy the Hölder condition on Γ.

2) The solution $f(z)$ is sought in the class of functions holomorphic outside D and vanishing at infinity. Its boundary value $f^-(t)$ satisfies the Hölder condition on any compact, which contains no

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