Strong jump inversion

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Abstract

© The Author(s) 2018. We say that a structure A admits strong jump inversion provided that for every oracle X, if X' computes D(C)'for some C ≅ A, then X computes D(B) for some B ≅ A. Jockusch and Soare (1991, APAL, 52, 39-64) showed that there are low linear orderings without computable copies, but Downey and Jockusch (1994, PAMS, 122, 871-880) showed that every Boolean algebra admits strong jump inversion. More recently, D. Marker and R. Miller (2017, JSL, 82, 1-25) have shown that all countable models of DCF0 (the theory of differentially closed fields of characteristic 0) admit strong jump inversion. We establish a general result with sufficient conditions for a structure A to admit strong jump inversion. Our conditions involve an enumeration of B1-types, where these are made up of formulas that are Boolean combinations of existential formulas. Our general result applies to some familiar kinds of structures, including some classes of linear orderings and trees. We do not get the result of Downey and Jockusch for arbitrary Boolean algebras, but we do get a result for Boolean algebras with no 1-atom, with some extra information on the complexity of the isomorphism. Our general result gives the result of Marker and Miller. In order to apply our general result, we produce a computable enumeration of the types realized in models of DCF0. This also yields the fact that the saturated model of DCF0 has a decidable copy.

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