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Degrees of categoricity and spectral dimension

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Abstract

© 2018 The Association for Symbolic Logic. A Turing degree d is the degree of categoricity of a computable structure S if d is the least degree capable of computing isomorphisms among arbitrary computable copies of S. A degree d is the strong degree of categoricity of S if d is the degree of categoricity of S, and there are computable copies A and B of S such that every isomorphism from A onto B computes d. In this paper, we build a c.e. degree d and a computable rigid structure Msuch that d is the degree of categoricity of M, but d is not the strong degree of categoricity of M. This solves the open problem of Fokina, Kalimullin, andMiller [13]. For a computable structure S, we introduce the notion of the spectral dimension of S, which gives a quantitative characteristic of the degree of categoricity of S. We prove that for a nonzero natural number N, there is a computable rigid structureMsuch that 0 is the degree of categoricity ofM, and the spectral dimension ofMis equal to N.

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Keywords

categoricity spectrum, computable categoricity, degree of categoricity, rigid structure

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