

DOI 10.1007/s10469-018-9495-4

Algebra and Logic, Vol. 57, No. 3, July, 2018

(Russian Original Vol. 57, No. 3, May-June, 2018)

TURING DEGREES IN REFINEMENTS OF THE ARITHMETICAL HIERARCHY

V. L. Selivanov^{1*} and M. M. Yamaleev^{2*,**} UDC 510.51:510.54:510.531:510.532

Keywords: *Ershov hierarchy, fine hierarchy, arithmetical hierarchy, Turing degrees.*

We investigate the problem of characterizing proper levels of the fine hierarchy (up to Turing equivalence). It is known that the fine hierarchy exhausts arithmetical sets and contains as a small fragment finite levels of Ershov hierarchies (relativized to \emptyset^n , $n < \omega$), which are known to be proper. Our main result is finding a least new (i.e., distinct from the levels of the relativized Ershov hierarchies) proper level. We also show that not all new levels are proper.

INTRODUCTION

Investigation of Turing degrees of Δ_2^0 -sets is a central topic in local degree theory. The structure of Δ_2^0 -degrees turned out to be quite rich and complex, and extending the local degree theory to higher levels of the arithmetical hierarchy leads to additional complications. In this paper, we consider some natural questions about such extensions related to one direction in local degree theory, namely to investigating degrees in the Ershov hierarchy.

This line of research was initiated by B. Cooper [1] who constructed a difference of computably enumerable (c.e.) sets which is not (Turing) equivalent to a c.e. set. Since the c.e. sets form the first level Σ_1^{-1} of Ershov's hierarchy $\{\Sigma_k^{-1}\}_{k < \omega}$ [2] and the differences of c.e. sets form its second level, we can ask about the possibility to extend Cooper's result to all levels. This was independently

*The work was carried out at the expense of the subsidy allocated to Kazan (Volga Region) Federal University for the fulfillment of the state task in the sphere of scientific activity, project No. 1.1515.2017/4.6.

**Supported by Russian Science Foundation, project No. 18-11-00028.

¹Ershov Institute of Informatics Systems, pr. Akad. Lavrent'eva 6, Novosibirsk, 630090 Russia. Kazan (Volga Region) Federal University, ul. Kremlevskaya 18, Kazan, 420008 Russia; vseliv@iis.nsk.su. ²Kazan (Volga Region) Federal University, ul. Kremlevskaya 18, Kazan, 420008 Russia; Mars.Yamaleev@kpfu.ru. Translated from *Algebra i Logika*, Vol. 57, No. 3, pp. 338-361, May-June, 2018. Original article submitted December 12, 2016; revised October 20, 2017.