УДК 381

CONTENT INTERACTIVITY AND CONTENT COMMUNICATION IN ENGINEERING OF ONLINE MATHEMATICS METHOD CLASS

(СОДЕРЖАТЕЛЬНАЯ ИНТЕРАКТИВНОСТЬ И ПРЕДМЕТНАЯ КОММУНИКАЦИЯ В ИНЖЕНЕРИИ ДИСТАНЦИОННОГО КУРСА ПО ОБЩЕЙ МЕТОДИКЕ ПРЕПОДАВАНИЯ МАТЕМАТИКИ)

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Abstract. In today's world, current revolutionary changes are associated with the intensive use of digital technologies in many spheres of human life, which democratize knowledge and access to open education. The ICT is increasingly implemented in the daily lives of individuals and the society. We are witnessing the formation of a new phenomenon - a global virtual learning community, which today includes more than one billion users. And the numbers continue to grow. Along with this, the market of online educational services is steadily growing. To meet the demands of the market, content development, content interactivity and content communication play important role in the engineering of online learning. In this paper, we will consider some of the approaches that will help to enhance content interactivity, such as cognitive visualization and other emerging techniques, for example, video streaming, screencasting, and gamification. We will also discuss different formats of content communication.

Keywords: content interactivity; content communication; visualization.

Introduction: From teaching to engineering of learning

Since 2000 the author has been studying the approaches to the use of Information and Communication Technologies (ICT) in education and distance learning. In 2001, he developed an open access web site "Visual Mathematics" (http://mourat.utep.edu/vis_math/visuala.html) and used dynamic cognitive visualization to represent solutions to mathematical problems and proofs. The website is used by the author in mathematics methods and mathematics classes at the University of Texas at El Paso, USA.

During the recent years the author has been developing and teaching hybrid/ blended (partially online) and distance (online) courses for pre-service and in-service training of secondary school teachers of mathematics. Analysis, modeling and designing of distance learning courses convinced the author that content and didactical knowledge are necessary but not sufficient for development of high-quality online courses. In addition, one needs to acquire a new type of knowledge that integrates content, didactics and engineering. Application of engineering approaches to didactics is called *didactical engineering*.

In this paper, the author shares his experience of practical application of didactical engineering of student learning through design of content interactivity and content communication in mathematics method class. The main emphasis of the paper is on understanding and designing the key features of learning experiences (e.g., objectives, content, assessment) through the use of Information and Communication Technologies (ICT).

Visualization as a means of content interactivity

Visualization is one of the few areas of research in education, whose relevance is continuously increasing over time in different subject domains including mathematics. It was relevant in 1957, when P. Van Hiele first presented the model of teaching geometry with a support for the development of student visual thinking (Van Hiele, 1986). The relevance of this problem sustained in the 1970-ies, when R. Skemp proposed the theory of conceptual scheme (Skemp, 1987). The significance of the visualization problem was emphasized in the 1990-ies by the publication "Visualization in teaching mathematics" (Zimmerman and Cummingham, 1990). The level of relevance of this issue is still dominating nowadays with its critical role in designing content interactivity for online learning (Sigmar-Olaf and Keller, 2005; Konate, 2008).

The direct application of the science of learning' findings in visualization such as "People learn better from words and pictures than from words alone" (Mayer, 2011: 70) to the practice of learning through recommendation "Add relevant graphics to text lesson" (ibid: 70) sounds invigoratingly simplistic. The meaning of visualization in learning is much broader yet complex than just 'adding graphics to the text'. Moreover, visualization plays a significant role in the engineering of learning via linking advances in the science of learning and the practice of using visualization in the classroom as shown in Figure 1.

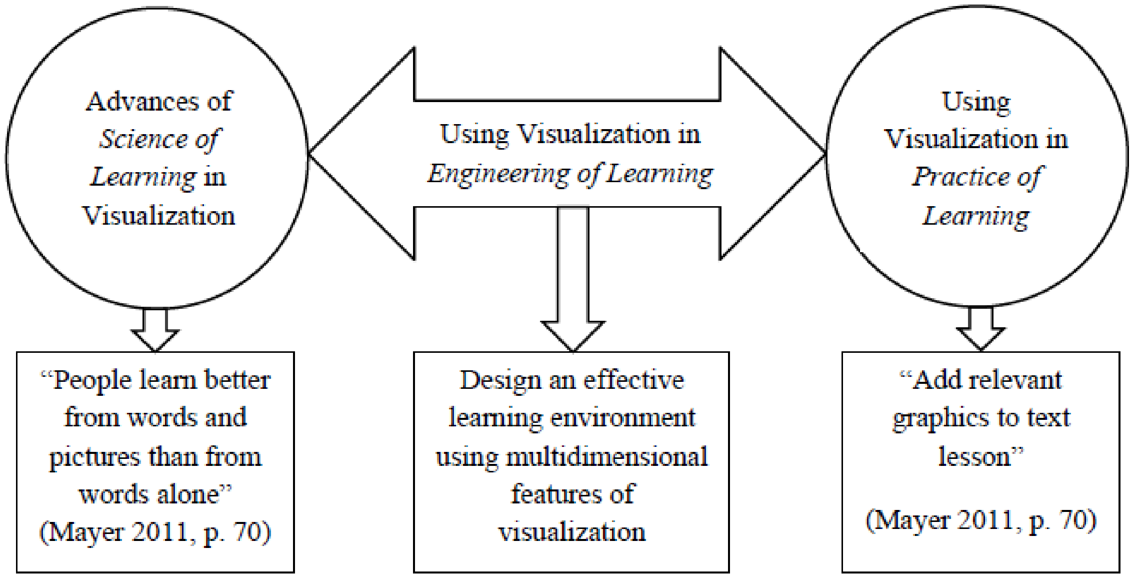

Fig. 1. Engineering of learning as a link between the science of learning and the practice of learning in using visualization

Visualization is a multidimensional construct that has several important characteristics. We will consider the following dimensions:

illustrative and cognitive visualization
static and dynamic visualization
passive and interactive visualization
isolated and connected visualization
visualization and multiple representations
academic and scientific visualization.

Visualization could be illustrative and cognitive. Illustrative visualization usually represents an answer to a low cognitive demand question such as: what is it? For instance, if one asks "what is an isosceles triangle?", a visual illustration of a triangle with two congruent legs would be a sufficient answer. Cognitive visualization goes beyond just illustration: it unpacks the meaning of the concept. For example, cognitive visualization is used to develop students' understanding of problem solving and proof in mathematics. Let say, we would like to visually represent the proof of the following theorem "Sum of interior angles of a triangle is equal to a straight angle". The proof of this basic theorem requires multiple steps, which are depicted in the cognitive visual representation (Figure 2).

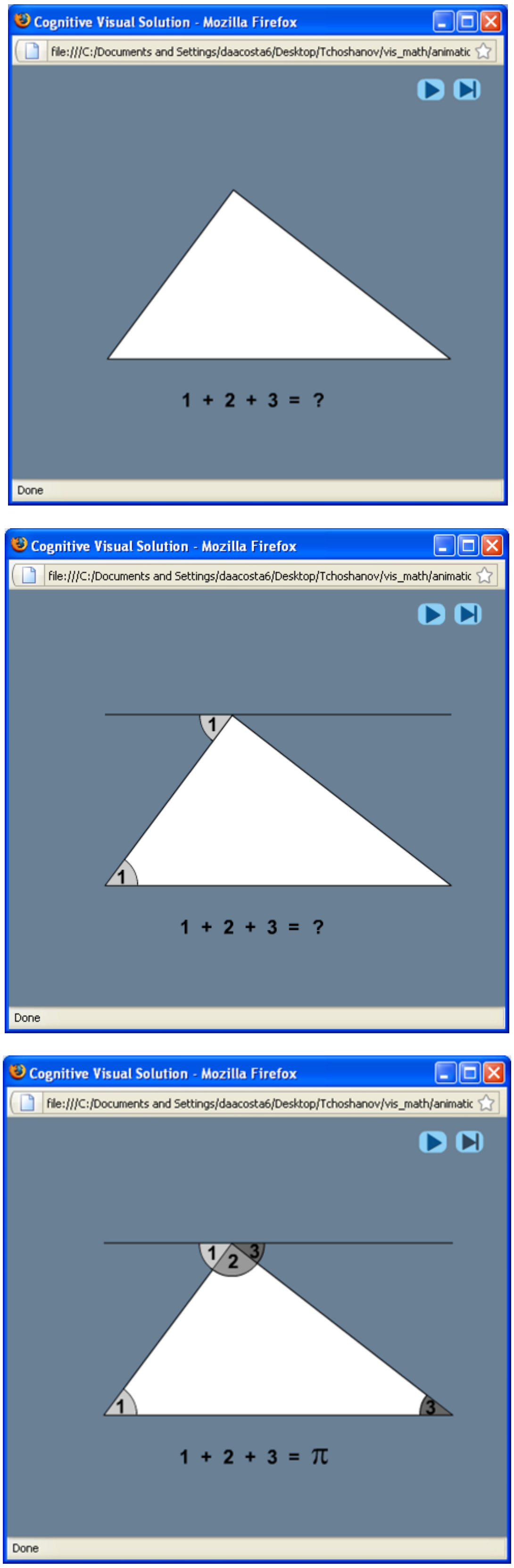

Fig. 2. Cognitive visualization of the theorem for sum of interior angles of a triangle

Visualization could be static and dynamic. Using the above example (Figure 24), we could represent the final step as a static visual image of the proof, or we could show the same proof in dynamics as a series of steps. Most of the visual proofs presented in a fascinating series “Proof without words: Exercises in visual thinking” (Nelsen 1993, 2000; Nelsen & Alsina 2006) are primarily static. Author’s open access website on

Visual Mathematics (http://mourat.utep.edu/vis_math/) consists of examples of cognitive dynamic visualization on various topics of mathematics (Figure 3).

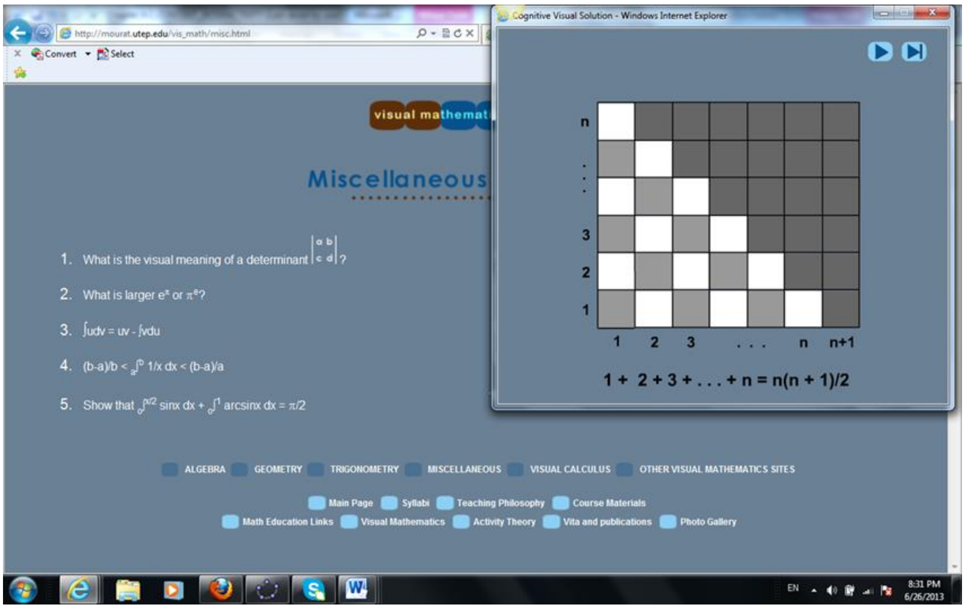

Fig. 3. Screenshot of the Visual Mathematics website

A dynamic visualization feature helps learners to develop their conceptual understanding and is intensively used in a variety of software packages such as Geogebra, Geometer's Sketchpad, Cabri, Mathematica, to name a few.

Visualization could be passive and interactive. Passive visualization requires little or no student involvement in the visualization process whereas interactive visualization allows students to manipulate certain parameters of the demonstration to better understand the concept. The open source Wolfram Demonstrations Project (Figure 4) presents interactive visual solutions using computer animations and applets to various mathematics and science problems where students can 'play' with the demonstration changing its parameters. For example, interactive visual solution to the problem of an area under cycloid presented in the Figure 26 has multiple benefits compared to an analytic solution: students can visually follow the trace of the cycloid, they can understand how the curve is produced, students can visualize the concept of the area under the cycloid, and finally, they can build conceptual understanding of why the area under the cycloid produced by a circle with a radius R is equal to $A = 3\pi R^2$.

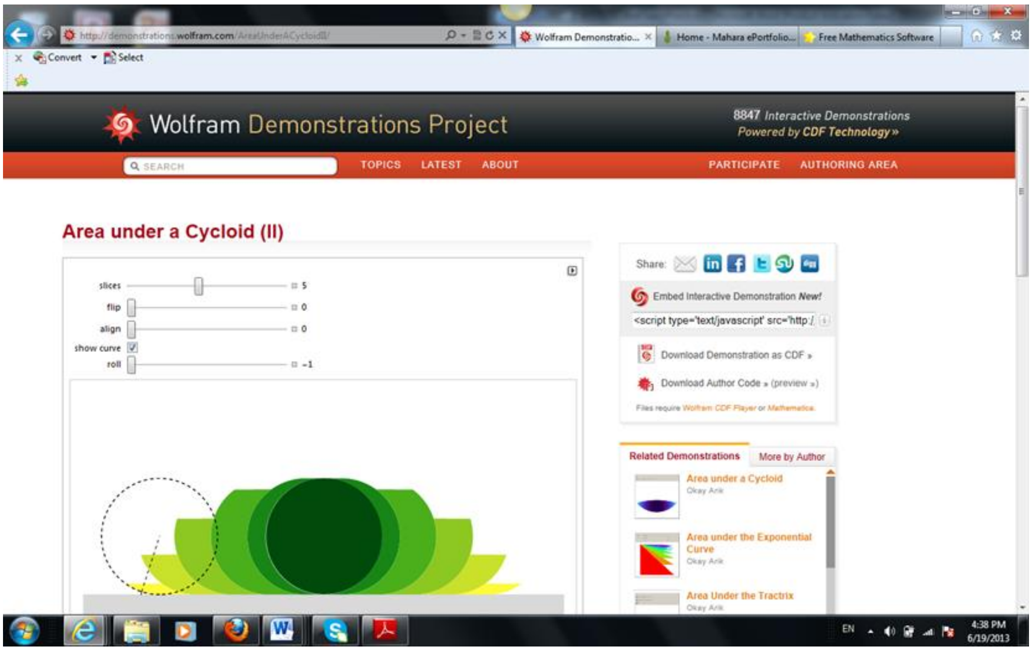

Fig. 4. Screenshot of the Wolfram Demonstrations Project

Visualization could be isolated and connected. Let us consider the following problem "The cookie monster sneaks into the kitchen and eats half of the cookie; on the second day he comes in and eats half of what remains of the cookie from the first day; on the third day he comes in and eats half of what remains from the second day. If the cookie monster continues this process for four days, how much of the cookies has he eaten? How much is left? If the process continues forever, will he ever eat all cookie?" The author used this problem in one of his graduate class with in-service teachers while discussing possibilities of early introduction of the infinity concept at the middle school level. In order to look for the solution, teachers usually start with making a table with the values given in the problem. Very few of them use visualization as a problem solving tool. After the class discussion on different methods of solving the "Cookie Monster" problem, they admit that the visual solution is the best one in developing students' understanding of the concept. One of the possible visual solutions is shown in Figure 5.

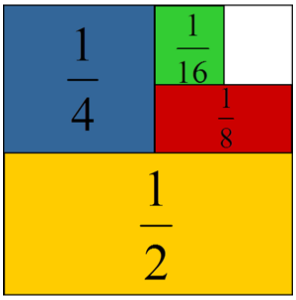

Fig. 5. Visual solution to the "Cookie Monster" problem

The discussion is further extended to other visual representations of the problem: teachers get engaged in considering the number line (using a bread stick instead of a square-shaped cookie), a pie model (using circle-shape crackers), or even cubic (using a 3D cubic-shape brownie) visual representation of solution. The teachers understand that within the same modality of visualization there could be multiple ways to represent the same concept. Most importantly, the teachers see the difference between an isolated visual image and multiple connected visual solutions for the same problem.

Visualization could be used as a singular mode and as one of the modalities in multiple representations. Using the same "Cookie Monster" problem, the teachers were able to synthesize multiple methods of solving the problem into the multiple representational diagram depicted in Figure 6. The visual solutions discussed above (e.g., number line, pie, square and cube models) are presented along with other multiple representational modalities (e.g., tables, graphs, equations, diagrams).

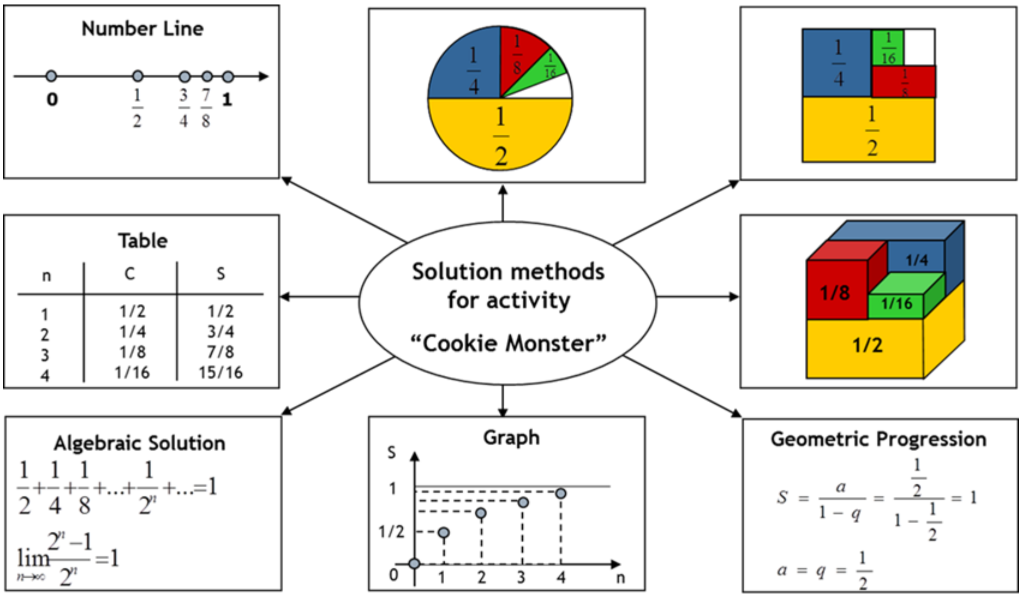

Fig. 6. Representational modalities for solutions to the "Cookie Monster" problem

Last but not least, visualization could be academic and scientific. The visualization examples presented above are all academic by nature because they are used to support student learning in a particular academic discipline. Scientific visualization is an interdisciplinary branch of science which is "recognized as important for understanding data, whether measured, sensed remotely or calculated" (Wright, 2007) and it is primarily concerned with visualization of three-dimensional phenomena in scientific research. Therefore, scientific visualization could be too advanced for students to grasp and understand. An important question here is how to get students motivated in searching for and appreciating the scientific visualization. For example, most of the high school and college students know what a 3-D cube looks like. However, many of them might be curious to know and surprised by what a 4-D cube looks like (see Figure 7: http://upload.wikimedia.org/wikipedia/commons/a/a2/Tesseract.ogv).

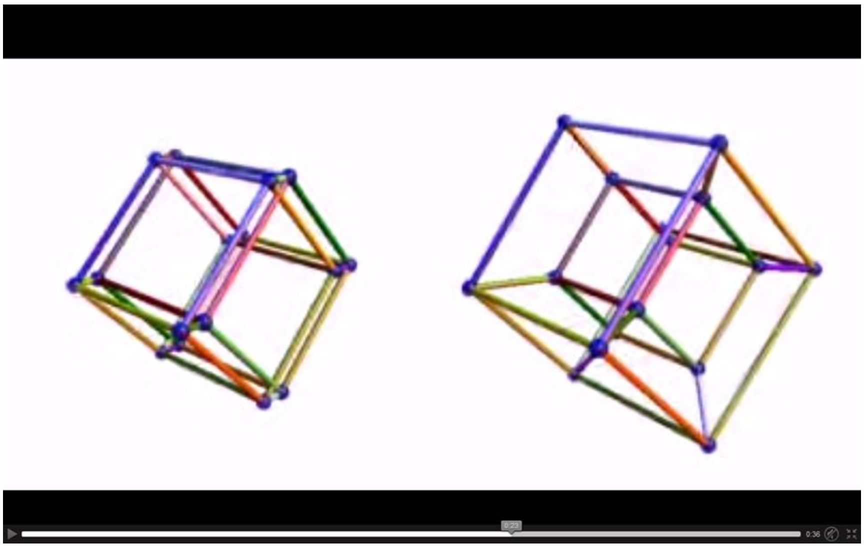

Fig. 7. Visualization of a 4-D cube: orthogonal (left) and perspective projection (right)

Addressing the visualization issue would be incomplete without considering the role of visual tools in the form of concept and/or mind maps to support student learning and understanding (Wycoff, 1991). The main purpose of a concept map is to engage students in making connections between concepts and procedures and expand students' understanding of a subject domain through a holistic perspective. An example of the concept map is presented in Figure 8 (http://www.svsu.edu/mathsci-center/uploads/math/gmconcept.htm).

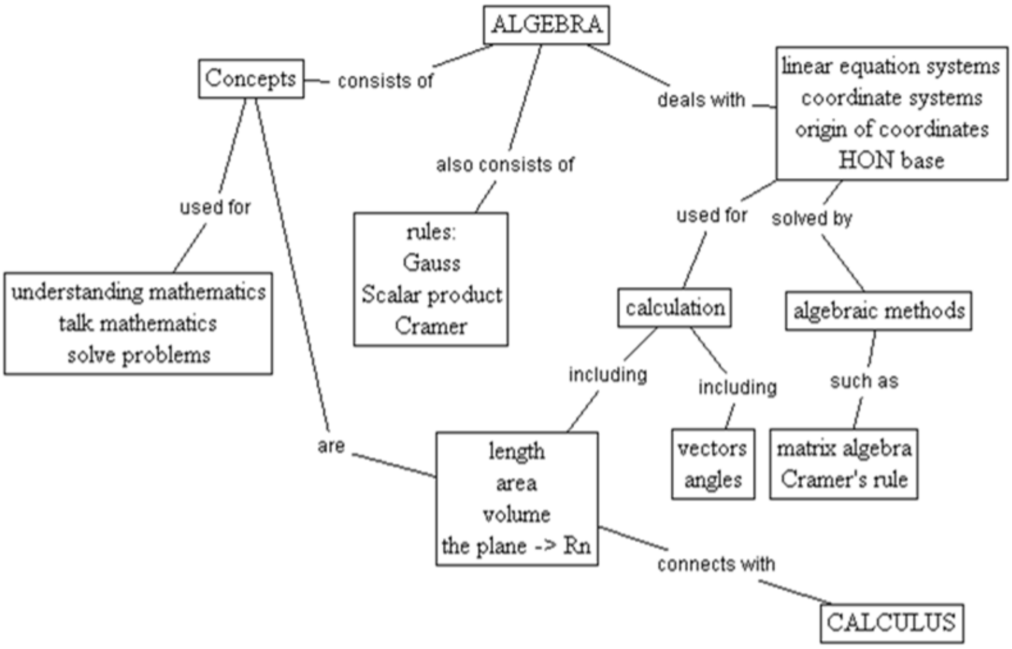

Fig. 8. Example of a concept map for Algebra

Video and/or media streaming and content interactivity

Video streaming is another widely used technique to enhance content interactivity. Video streaming helps learners to understand complex concepts that are not quite convincing to explain with plain text and graphics (Klass, 2003). Video streaming is particularly important for online learning due to its distinct interactivity component. Incorporation of multimedia including video streaming can improve the learning process as students see the concepts and ideas in action (Michelich, 2002). "In addition, a moving image can help students visualize a process or see how something works. Video can take tacit information or knowledge that may be too difficult to describe in text into an articulate, vivid description through the use of images" (Hartsell and Yuen, 2006: 32). Video streaming can evoke emotional reactions and increase student motivation. Furthermore, streamed videos can be accessed by students at any location that has an Internet access (such as library, home, café) and at any time. Another advantage is a student choice over priority and sequence of video materials to be observed on-demand. The true advantage of video streaming is an opportunity for self-pacing online learning: students are in charge of starting, pausing, skipping, and reviewing the media material. Among major limitations in implementation of video streaming in online learning could be resources, support structure and personnel training, since "it is difficult to sustain streaming video in academic institutions because of limited access to technology and knowledgeable experts who can assist maintaining and developing media streaming" (Shepard, 2004). There are ample opportunities for video and media streaming offered by variety of educational sources such as Discovery Education (http://streaming.discoveryeducation.com/), National Geographic (http://video.nationalgeographic.com/video/), NBC Learn (http://www.nbclearn.com/portal/site/learn/) and many other resources. An example of NBC Learn media streaming site on "Science of NHL Hockey" is presented in Figure 9.

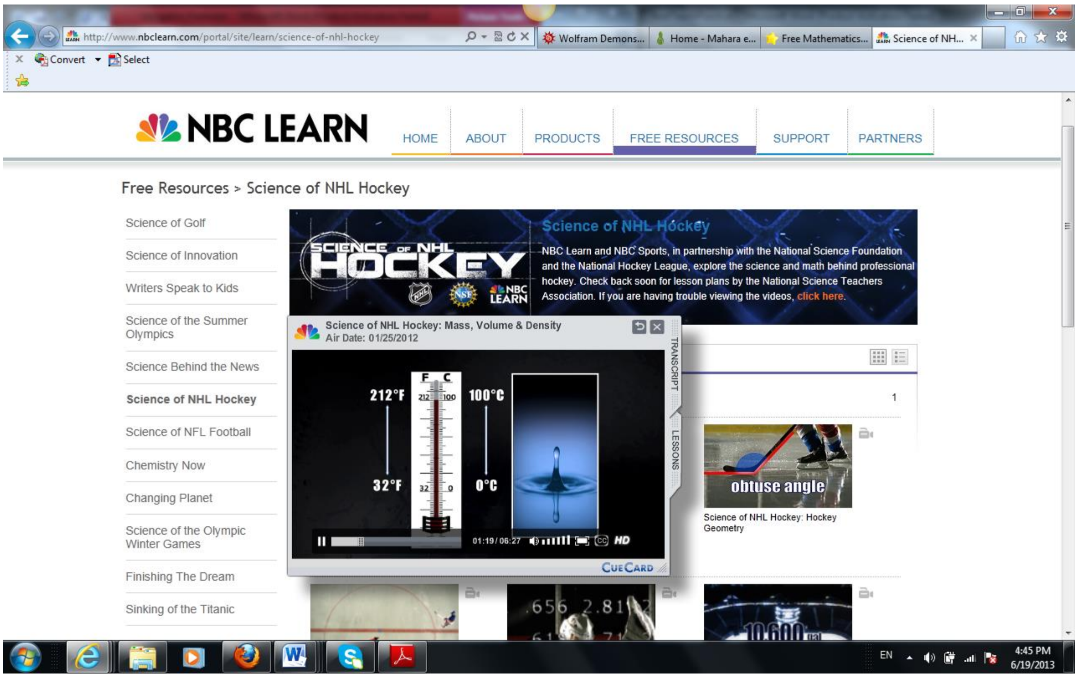

Fig. 9. Screenshot of the NBC Learn media streaming resource

Screencasting is a technique of creating dynamic and engaging content through digital video and audio recording of a computer screen while developing tutorials and demonstrations. Screencasting could also be used for digital storytelling and narrated presentations with a variety of media (e.g., video clips, pictures, graphs, and animations) imported into it. There are multiple advantages both for students and instructors in incorporating screencasting in learning. Screencasting is an effective tool that helps teachers to explain difficult concepts and allows students to learn a sequence of steps in performing a certain procedure, working on a task and solving a problem. Similarly, with video streaming, students can watch a screencast anywhere and anytime. Moreover, students can review any part of the screencast, pause, rewind, and repeat it as needed, which creates an effective learning environment for self-paced learning. Screencasting can be used to fulfill a variety of learning objectives, including but not limited to topic introduction, overview of the concept, discussion, and skill practice. Screencasting is widely used by open source repositories, such as Khan Academy (Figure 10), to provide opportunities for "*flipped classroom*" activities (Bergmann & Sams, 2012) when students watch teacher's screencast lecture as a homework and use class time for discussing difficult topics and challenging problems, working on projects, activities, etc. In order to produce a quality screencast, teachers need to have screencasting software (e.g., Webinaria, Jing, Screencast-o-Matic) and the screencasting tools such as microphone (for narration), webcam (for video), digital tablet or touch-screen with stylus (for drawing), etc. "The most obvious drawback of screencasting is that it is not interactive. Although some lessons lend themselves to fixed demonstration, others do not and should not be taught with screencasts… Simply recording the instructor's screen during a class session can be an inefficient way to transfer information" (ELI, 2006).

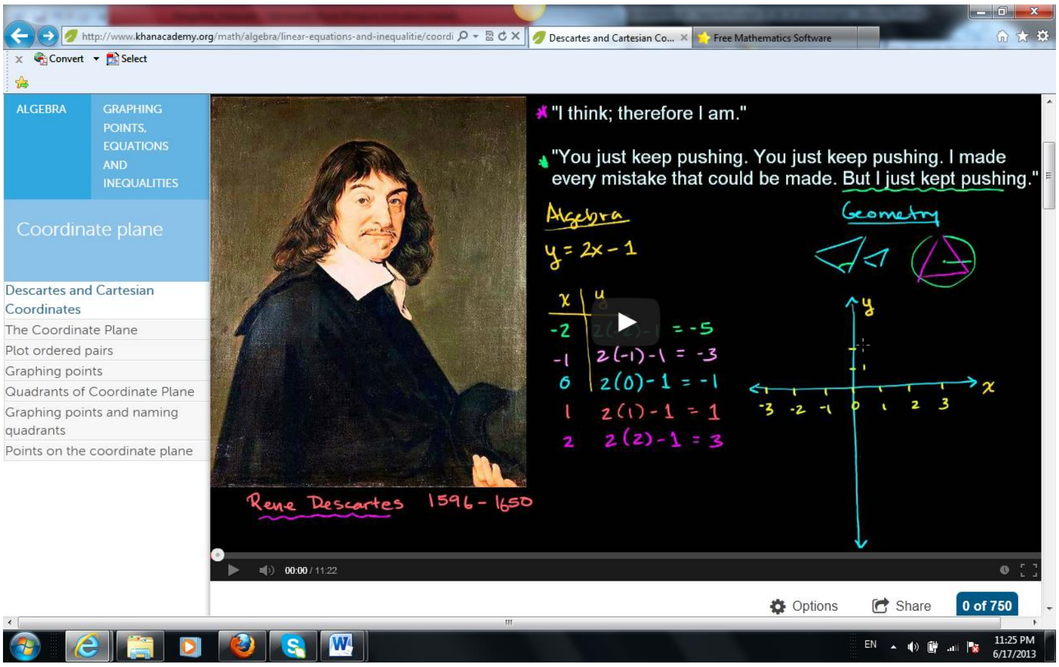

Fig. 10. Screenshot of the Khan Academy use of screencasting

Gamification, game-based learning, or game-informed learning are the names for the emerging phenomenon in education - “using game-based mechanics, aesthetics and game thinking to engage people, motivate action, promote learning, and solve problems” (Kapp, 2012: 10). As a pedagogical approach, gamification is constructive by nature and built on the elements of multiple intelligences’ theories, situated learning, experiential learning and the activity theory. Gamification allows students to learn and experiment in a non-threatening environment, supports learning by doing through social interaction and collaboration. Gee (2007) emphasizes that “a good instructional game would pick its domain of authentic professionalism well, intelligently select the skills and knowledge to be distributed, build in a related value system as integral to gameplay, and clearly relate any explicit instructions to specific contexts and situations”.

Well-designed gamification has multiple benefits including but not limited to providing authentic learning context and activities, multiple roles and perspectives in co-construction of knowledge as well as encouraging scaffolding and integrated assessment. An example of gamification is “Function game” where by inputs and outputs you have to identify a function (Figure 11).

Along with benefits there are some limitations to the gamification approach. The *content should be a major driving force for designing game-based learning.* Unfortunately, gamification based on the quiz-and-reward format only is not the most effective way to engineer learning and motivate students. Well-designed gamification supports high cognitive demand content and focuses on students’ understanding and reasoning more than just memorizing facts and procedures. Another critical consideration in gamification has a natural and seamless connection between the game and the learning: the game improves the learning and the learning supports the game. A well-designed gamification also carefully balances content, learning and assessment.

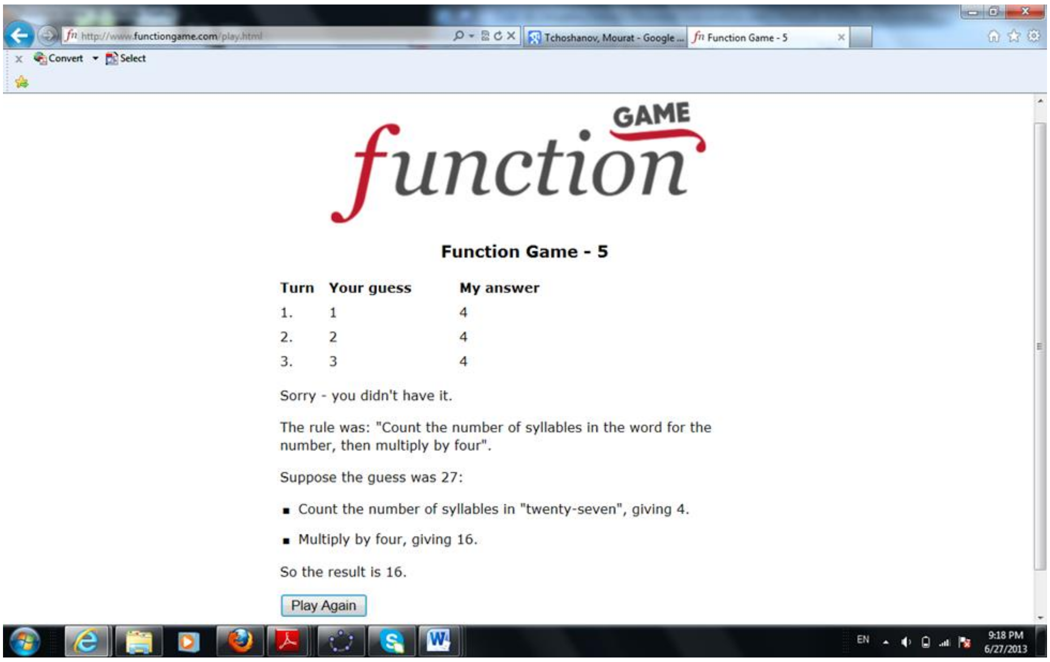

Fig. 11. Screenshot of the "Function game"

Content communication

Along with the content development and content interactivity, promoting and facilitating content-focused communication between the instructor and students is critically important to the success of the course whether it is face-to-face, hybrid, or online. With regard to distance learning, the content communication is an essential point of distinction between truly effective online course and poorly designed old-fashioned correspondence course. The content communication within an online course could be organized in individualized and/ or group-based format. It also could be synchronous and/or asynchronous. Regardless of the format, the communication is a key to creating and sustaining an effective learning environment in the course.

In order to initiate and encourage communication between students, it is helpful to provide an opportunity for students to introduce each other at the beginning of the course. There are various tools available to support individualized communication such as texting, e-mailing, using Skype, FaceTime, Facebook, Twitter, etc. Instructor may choose to schedule phone or Skype conversations with individual students in an online course during virtual office hours which should be posted in the course syllabus. As an instructor of the course, you may also interact with individual students via text messaging and e-mailing. Another form of virtual communication with individual students is using Skype and/or FaceTime that enables face-to-face interaction by video as well as by voice. Instructor may also use social networking tools such as Facebook and/or Twitter to communicate with individual students as well as with the groups of students and the whole class through posting messages, blogs, and other ways of promoting communication.

Group communication and discussions are equally critical for the online course as individual communication. Various learning management systems offer multiple channels for group communication such as chat rooms, different modifications of discussion boards (e.g., Contribute, WebEx), collaborative document sharing and editing tools in real time (e.g., Google Docs, CampusPack). These virtual tools allow students and the instructor to engage in a text-based synchronous group conversation and discussion for various purposes including but not limited to the review sessions for major course assignments, to discuss group projects and presentations. Instructors have preferences in using particular tools for the group communication. Let us share an example of using the Blackboard discussion board to promote group communication in a content-specific topic. The graduate class of in-service middle school teachers was assigned to read the chapter on rational numbers and take a test. One of the questions in the test is below:

"Which of the statements below is true?

a) $2.4999\ldots < 2.5$

b) 2.4999… = 2.5

c) 2.4999… > 2.5

d) Cannot be determined given the above information.

Explain your answer."

The level of complexity of this item is determined by its connection to the fundamental idea of duality. Most of the class participants felt unfamiliar and challenged by the question posted in the assignment. Some of the students who selected the answer "a", e-mailed the instructor expressing the confusion. The most trivial solution to this situation is that the instructor could simply provide a correct answer to 'avoid' discussion on the challenging concept. However, this option would significantly limit student learning. The instructor (his signature in the Table 1 is represented as *mt*) decided to provoke the whole class discussion using the Blackboard. As depicted in the table, the discussion consists of four major stages:

1. Provoke: instructor selects a provoking question and invites participants to the discussion; the instructor monitors student responses and provides clarification.
2. Sustain: instructor capitalizes on students' reasoning to require further exploration.
3. Evaluate: instructor asks students to explain and evaluate the solution.
4. Synthesize: instructor brings a closure to the discussion.

The table also includes discussion actions and discussion context to illustrate the complexity and challenges of purposefully-orchestrated discussion in supporting student learning.

Table 1. The fragment of content communication via discussion board

Discussion stage	Discussion action	Discussion context
Provoke	Instructor selects a provoking question and invites participants to the discussion	*Dear All, one of the participants had difficulty understanding the problem 5 on Chapter Test #3. The student wrote: "I don't understand why my answer (letter A) was incorrect. 2.4999... has to be smaller than 2.5". Do we have people answering this problem differently? Share your responses, please. mt* *Dr. Tchoshanov, I agree with the student, due to the construct or the limited information though of the quesiton*[1] *regarding the answer responses. I understand what the student is thinking. 2.4999 is smaller than 2.5, unless you estimate the value (though this was 'not' indicated as an approximation). They are "virtually" the same, but they are not, there is a difference which is miniscule. There is no way we could view the difference. For example, in measurement all measurements are approximations, a measurement of 2.5 and 2.4999... would be virtually the same, if you are in 'approximation.' Techinally, it is smaller value even if the value is a miniscule in difference. Brianna* *Brianna, I also agree with you. Mathematically, I think 2.4999…. is less than 2.5 because there is a very small difference in between these numbers. Also, we can say 2.4999… is approximately equal to 2.5. I do not think 2.4999…. is equal to 2.5. If we see this problem through student's point of vie, 2.4999… is equal to 2.5. Because, in a number line, 2.4999…. is very close to 2.5. We teach them to round to the nearest number in the number line. Pat* *When I answered this question I was picturing a number line which in that case the 2.49999 is smaller than 2.5, but then I second guessed myself thinking should I round up to the nearest tenth? If so, the two numbers would be equal. I guess as you say it all deals with the approximity of your numbers. Enrique* *I too think that if you look at it in a technical and mathematical way, 2.4999 is*

[1] Students' grammar and style are intentionally left unchanged.

Discussion stage	Discussion action	Discussion context
		literally smaller than 2.5, but if it is being compared through the form of approximation then they are the same. Depends on how you look at it. Radhika *Radhika, I completely agree on your thoughts, it really depends how you are viewing the contexts of this problem. I do not believe there was sufficient amount to answer if greater than or equal. It does depend on how you see it, I do not think it incorrect. I put D. for the answer (I view things in a technical light) since all the above answers is plausible, if your counting the approximations or not. Good point. Brianna*
	Instructor monitors student responses and provides clarification	*However, the problem didn't ask for rounding or approximation. mt*
		I think we can all make a strong point for every answer choice there was, but the question did not state if this was an approximation or not, so i read the question in its most literal definition and chose the answer the was most correct, I also chose A. Jaime *I agree that it really depends on how you view it which is why I also chose D on this question. I can definitely see why A looks like a good answer because really it could be true but I too think it depended on how you viewed the problem which is why I ultimately chose D. Samantha* *When I answered this question, I chose to think of it in terms of fractions. For instance, 1/3 can be represented physically. But if you put it in decimal form, 1/3 is the same as 0.3333.... Then I thought to myself, is this number less than 0.34? Yes! I can represent both. So to me 2.4999.... is less than 2.5. I as well do not understand why a is wrong. I went through the reading as well as searched the web and looked in my old math texts. I did not find anything contradicting my idea. Ann*
Sustain	Instructor capitalizes on students' reasoning to require students exploring further	*… let me provide you with a counterexample to sustain the discussion. Ann uses a very convincing argument saying "1/3 is the same as 0.333..." If we accept Ann's argument, then let's do the following:* *a) lets multiply both sides of 1/3 = .333... by 3;* *b) (1/3)x3=(.333...)x3* *c) 1=.999...!* *Share your insights on 1=0.999..., please. mt* *Dr. Tchoshanov, lets consider the inequality that we use for domain and range of a function (introduction of function in Algebra 1) with a graph using closed and open circles. For example, the domain of a graph of a function with an open circle at x=1 extend to the negative infinity is -∞≤x<1. Even though the function is very close to x = 1, the domain is not -∞≤x≤1. Thank you. Rick* *Rick, very valid point. Thank you. The question is how do we connect the two ways of reasoning about the same concept? mt* *… I asked a middle school math teacher and she didn't know. Then I asked an engineer and he sent me this email:* *Debbie,* *2.49999... = 2.5. To prove this, assume: 10 * x - x = 9 * x, so: 24.9999... - 2.49999... = 9 * 2.4999...Considering that 0.0999... will cancel in the*

Discussion stage	Discussion action	Discussion context
		*subtraction, then: 24.9 - 2.4 = 9 * 2.4999... Simplifying: 22.5 = 9 * 2.4999... Dividing by 9: 2.5 = 2.4999... QED It did make sense. We know that simply substituting numbers didn't necessarily make something true. Here is a case where you could try simple numbers like two or three and the final numbers would be the same, but if you substituted 2.4999..., it would come out as 2.5 on one side and 2.4999... on the other. However, the expression still holds even though there is a case where substituting doesn't work. This is a very interesting problem and I'm curious to see what others will say about it. Debra*
Evaluate	Instructor asks students to explain and evaluate the ‘engineer’ solution	*Debra, I appreciate you researching this problem and getting an engineer involved. I think he has a solution to be discussed further. Let’s call it the ‘engineer’ solution and ask everybody to share their insights on this. Post your reaction on the ‘engineer’ solution, please. mt* *Here is my attempt to go against the engineer just to be difficult. The problem states 2.5 equals 2.4999… I think there is a difference of saying "exactly 2.5" and "infinitely close to 2.5". We can say that 2.4999… may have a limit but it will never be reached because it does on forever, so in reality there is a difference between both. Depending on your calculator 2.49! does not equal 2.5! If we consider this in a real word application and have two runners one a time of 2.49 sec and one with 2.5 sec who would be considered the winner? I think infinity is a concept and not a number, it’s like saying 1/infinity = 0 you cannot divide a number by a concept. Jaime* *Hi Debbie, Thanks for posting the engineer's solution. I went from step to step, and realized it did make sense. I never had this mathematical training as most engineers would receive. A lot of my education, in my undergraduate work has been fully in the Liberal Arts category. It keeps reminding me of DNA how the match of 99.9999...% is essentially a complete or 100% match. It makes sense, after this supplemental solution. Again, it was very interesting viewing this! Brianna* *This question is really bothering me. My answer was A, because the question was very straightforward: “Which statement below is true?” And it is true that 2.49999... < 2.5. It does not matter how many 9’s we add to the 2.499.... it will never reach 2.5, it will always be smaller than 2.5. I also have talked to some people, a PhD mathematics student told me that of course, 2.499 is smaller than 2.5, but that it will also depend on the context. Looking at the context of the question, my answer is still <. As an engineer myself, I know how critical is to work with decimals. Juan* *I actually enjoy reading the lively discussion this problem has created. I think it helped me see "proof" in a new way, and it was a good extension of our previous discussions. I believed the instructor also pushed us to come up with our own understanding of the challenging problem. Joanna*
Synthesize	Instructor brings a closure to the discussion	*Dear All, this was a thought provoking discussion and, most importantly, it exemplified the convincing a skeptic strategy that we have discussed last week. Let me synthesize the discussion.* *Juan made a good point that the solution to this problem “depends on the context.” Pat earlier mentioned that “… mathematically, I think 2.4999…. is less than 2.5 because there is a very small difference in between these numbers.” At the same time, Debbie presented the ‘engineer’ solution to the problem that convinced some of the participants: 2.4999…=2.5. Extending further, Jaime argued that “there is a difference of saying "exactly 2.5" and "infinitely close to 2.5.”*

Discussion stage	Discussion action	Discussion context
		Thus, throughout the discussion we were looking at the same problem from the two distinctly different lenses: (1) the 'process' view (e.g., 2.4999… <2.5), and (2) the 'object' view (e.g., 2.4999… = 2.5). In mathematics education, this phenomenon is called 'process-object duality'. We will be further unpacking the idea of duality in our forthcoming discussions. *Greatly appreciate everybody's input into this intellectually challenging yet engaging discussion. mt*

A well-designed and seamlessly implemented content interactivity and content communication significantly contribute to the effectiveness of learning environment in face-to-face and online education.

Conclusion

In today's world, current revolutionary changes are associated with the intensive use of digital technologies in many spheres of human life, which democratize knowledge and access to open education. The ICT is increasingly implemented in the daily lives of individuals and the society. We are witnessing the formation of a new phenomenon - a global virtual learning community, which today includes more than one billion users. And the numbers continue to grow. Along with this, the market of online educational services is steadily growing. This creates a domino effect: along with the transfer of many university disciplines, including teacher education courses to the online format, there is a need to revisit the training of school teachers. Instead of the traditional teacher training, the focus is shifting toward a new type of training for teachers who can work in the digital age, with high demands on teachers' knowledge and ability to engineer an effective online learning. Moreover, in the digital era a teacher is not just an online tutor, s/he becomes an analyst and manager of informational resources, a designer and a constructor of courses, modules, and lesson fragments using interactive multimedia tools.

The 'engineering of learning' paradigm places a critical emphasis on the development of teachers' engineering design thinking. The development of teacher-engineer's design thinking is a complex process based on the advancements of the learning sciences. It involves the following key competences:

1) the design of learning objectives: to create outcome-based, technology-enhanced learning environments that enable students to set their own learning objectives, monitor and assess their learning progress;

2) the engineering of content: to develop interactive content and relevant learning experiences through the selection and design of tasks, problems, projects, and activities that incorporate digital tools and ICT resources to promote student learning and creativity;

3) the design of assessment: to select and develop authentic assessments aligned with the learning objectives and content, and to use assessment data to improve teaching and promote student learning.

In order to respond to the challenges of the digital age, didactics itself needs to be re-conceptualized. This re-conceptualization has a clearly defined vector. Modern didactics is moving towards strengthening its "engineering" functions - didactical engineering. The development of didactics in the direction of the didactical engineering offers new opportunities for further understanding of learning and teaching in the digital age and creating effective learning environments in an emerging global learning community.

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