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QUALITATIVE STUDY OF SECONDARY MATHEMATICS TEACHERS' NOT-KNOWING WHILE SOLVING GEOMETRIC REASONING TASKS

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Abstract. Not-knowing is an underexplored concept defined by an individual's ability to be aware of what they do not know as a means to plan and more effectively face complex situations. This qualitative study focuses on analyzing students' ability to express their "not-knowing" while completing tasks and reflecting periodically. It becomes evident rather quickly that these students have difficulty expressing their not-knowing. Through transcription analysis, reflection coding, and interviews, four recurring themes emerge that could possibly determine why students have difficulty expressing their not-knowing. These four themes are deflection, student pressure, heuristic sense, and fractured knowledge. Each one of these themes will be discussed followed by a conclusion of their overall importance in relation to a students' ability to express not-knowing.

Keywords: Teacher not-knowing, secondary school mathematics, geometric reasoning.

Introduction.

Not-knowing is the first step to understanding, carrying an important value in learning. Mason (1999) claims, "awareness of knowing and of not knowing is crucial to successful mathematical thinking." Tahta (1972) uses not-knowing as a means to describe the algebraic process of finding what we do not know with the use of what we do know. Shah (1968) quotes an ancient wisdom according to which not-knowing is a critical state because, from it, knowing can follow. However, a little research has been done to understand the not-knowing phenomenon. In this study, we are examining the following research questions: how do students express what they do not know? and, what challenges do they face in externalizing the not-knowing?

Framework.

The theory of unconscious thought (Dijksterhuis & Nordgren, 2006) is closely related to the main construct and the unit of analysis of this study. Addressing the theory, Funke (2017) elaborates, "the basic idea is that the quality of decision-making depends on conscious and unconscious thought simultaneously. The term *conscious thought* is understood to mean a mental state that encompasses a person's rational awareness, whereas the term *unconscious thought* refers to the underlying influence, of which one is typically unaware and which has an impact on one's behavior. Unconscious thought takes place when conscious attention is directed elsewhere. Unconscious thought tends to outmatch conscious thought, especially in complex and untransparent situations (Funke, 2017).

Analysis of conscious thought, such as that of not-knowing, can be considered in a student's choice-making while problem-solving.

Being aware of one's own not-knowing is a spark that could potentially ignite inert knowledge. Renkl, Mandl, and Gruber (1996) suggest that "the problem of inert knowledge is surely of major educational importance". If awareness of not-knowing is a potential solution to inert knowledge, it is a topic that needs to be further explored.

Carlson and Bloom (2005) claim that a good problem solver exhibits flexibility as well as powerful math related processes to arrive at their solution. They also state that those who solve the problems do not

solely rely on heuristics. The awareness of not-knowing opens the doors to becoming more flexible at solving problems while becoming less dependent on heuristics.

Simon (1996) details "transformational knowledge" as a way to assess a given situation and select the best possible outcome. This line of thinking could potentially be achieved if individuals become aware of their not-knowing and use it as a means to understand a situation.

Therefore, not-knowing could be considered as the first step to effective problem-solving. This is one of the key motivations of the study, as it aims to uncover how not-knowing can help students understand their own thinking and more effectively learn in the classroom.

Methodology.

This qualitative study focused on students' articulation of not-knowing and challenges they faced during this process. Ten students were selected for the study at a university in the southwestern border region of the United States. These students were enrolled in the course on Geometric Reasoning, which focused on problem-solving using a tangram (a seven-piece puzzle) to construct squares with a different number of pieces. This setting was used to collect data and analyze student externalization of their not-knowing. Data sources consisted of audio recording, reflections, and interviews described below.

Recordings and Transcriptions.

At four different points during the course, students paired up and audio-recorded each other depending on who was attempting to create a given square using the tangram pieces. The task was to attempt to complete the square using the given pieces while vocally expressing what they knew and did not know at that specific moment. First, the participants recorded the seven-piece attempt, followed by a second attempt at the seven-piece, then the six-piece, and finally the five-piece.

The audio recording of the participants while attempting the task should bring forth their thought process while focusing on voicing what they knew and didn't know at the time. This was the reasoning behind the audio recordings, as it was thought to be a way of encapsulating students' not-knowing as a data source for analysis.

Reflections.

Two major reflections were assigned: post-activity reflections and post-lesson planning reflections. These reflections contained three and five questions respectively. The participants were tasked to reflect on certain ideas discussed as well as their knowing and not-knowing. Since these reflections asked the participants what they didn't know directly, their answers could be used to analyze how they expressed their not knowing.

Interviews.

Two students out of ten were selected for semi-structured interviews conducted at the end of the course to analyze student reflective thoughts on the course. The interview consisted of thirteen questions focused on extracting student not-knowing in a reflective fashion, which was audio recorded. The audio recordings were then transcribed and analyzed.

Data analysis.

In order to analyze the transcriptions, reflections, and interviews, meaning coding, meaning condensation and interpretation techniques were used (Kvale & Brinkmann, 2009) as main methods of analysis. The first set of analysis consisted of transcriptions from the participants' attempts at creating the seven, six, and five-piece squares using the tangram while voicing their knowing and not knowing. There

were two methods of analysis used for this part. First, the transcriptions of each individual participant were separated and analyzed to examine how well the participant expressed their not knowing at different points in time during the course. Second, an analysis of the transcriptions as a whole was conducted, in an effort to encompass key similarities between them. The second set of analysis was two reflections in which students reflected on what they knew and did not know at the given time based on the course. Meaning coding technique was used to interpret their ideas and make connections to their transcriptions. Finally, the last part of the analysis included the two interviews. The interviews are meant to provide a closer look into the thoughts of the two participants. The questions are aimed at evoking not-knowing reflectively and active not-knowing while the interview is being conducted. Meaning interpretation and meaning condensation were the primary tools in analyzing this data to make connections between other forms of not-knowing expressed throughout this study.

Findings.

The data analysis clearly demonstrated that the participants had difficulties expressing their not-knowing. The analysis shows that there are several recurring statements made by the participants. These statements were categorized and the following four major themes emerged that will be described below.

Deflection.

Deflection of not-knowing can be identified as an avoidance of challenge when an individual shifts the focus of their not-knowing somewhere else besides themselves. Below is an example of the participant's response, which demonstrates deflection of not-knowing:

Student: I know as far as formulas and all that they are not going to help me at all. It's more of a pattern thing and if students tried to do this it would be the same thing for them…

The participant makes it a case that she believes formulas will not help her at arriving at a solution, inferring that she does not know how to arrive at the solution. Her thought of not-knowing takes a shift stating that if students tried, it would be the same for them. Why is it that when asked about her own not-knowing she deflected? Instead of being aware of her not-knowing as a first step to finding the answer, the individual deflects to what she believes others don't know. A total of five out of the ten participants deflected at one point or another throughout the analysis.

Pressure.

Participants demonstrated pressure through direct vocalization, frustration, or sense of urgency. Every one of the participants demonstrated pressure at different points during the transcription and reflection sections. Look at these two statements below:

Student 1: Makes no sense to me. Jesus Christ… Ok, so it doesn't make sense... Jeez. It's almost something. Oh, God. Can I make a rectangle?

Student 2: This is ridiculous (laughs). Putting these squares together. It's a lot harder than I thought it would be. It's destroying my idea of what a square is… Maybe if I … no. Oh my God, this is so much harder than I thought it would be. I think you did that. Then we can put a little triangle here. I think that's what you had, isn't it? No. Argh, this is so frustrating.

These two students demonstrate pressure, which may be a factor impeding awareness of not-knowing. The first student demonstrates clear frustration throughout his thought process at his inability to make sense of the situation. The second student directly expresses her frustration, derived from expecting the task to have been easier than expected.

Lack of heuristic sense/trial-and-error.

Every single participant attempted the first task through trial and error as demonstrated by his or her transcriptions. We hypothesize that in seeking the solution, students may become "tunnel visioned" in the process of trial and error, clouding their awareness of not-knowing. Below is part of a transcription, where a participant demonstrated "tunnel vision."

Student: I am going to start with the parallelogram just because it has the oddest shape. And ... I'm going to try to make the sides even, and I'm just trying to add from there but it doesn't fit. So I don't know how to get the sides to be straight without having any leftovers. Ummm ... ok. No, I'm going to start again. I'm going to start with the big pieces now. I'm going to put the two triangles together. Okay, I'm going to put the two big triangles and try to make everything fit in the middle. Okay, so I'm putting some and they don't fit but I'm kind of getting the shape, kind of not.

Most of the participants, even with new knowledge, stuck to trial and error to the very end. Perhaps, inability to evoke awareness of their not-knowing was a factor that led to not solving the tangram.

Fractured knowledge.

Fractured knowledge is present when a participant may have knowledge gaps within the given topic, have misunderstandings of said topic, or simply lack the prior knowledge required for the given topic. We argue that if an individual has fractured knowledge, it will directly impede their ability to use not-knowing as a means to gather knowledge that simply is not there or is "fractured." Below is a representation of a participant with fractured knowledge:

Student: Ok, so that might be too long. So, I think it has to be smaller than 2 and square root 2. Maybe it can be 3? I will try for 3.

While attempting to find the side length of a square that must be constructed, the participant makes the revealing statement above. There is a clear misunderstanding of the number $2\sqrt{\mathbf{2}}$ ("two square root of 2") in comparison to the number 3. The participant believes that 3 is smaller than $2\sqrt{\mathbf{2}}$ and carries on without a second thought, guiding her down the wrong path.

These four themes frequently emerged throughout participant transcriptions, reflections, and interviews. Even though some of these themes emerged less than others, they all hold importance, as they reflected challenges in participants' ability to express not-knowing.

Discussion and conclusion.

Individuals "deflecting" their not-knowing should not come as a surprise, as not many individuals are fond of admitting their lack of knowledge or understanding. Nevertheless, the presence of deflection in the analysis shows how someone may shift their not-knowing onto someone else, rather than accepting their not-knowing and use it as a means to find a solution to a given problem.

The theme of "pressure" did not come as a surprise either. It may be closely related to Krashen's affective filter hypothesis (1985), which details students' abilities to learn based on what other thoughts might be on their mind. In this study, we saw pressures caused by time, frustration, and even fear of being judged by others.

The "lack heuristic sense" theme observed in the study is one deeply rooted in students' minds. Since one of the most fundamental heuristics is trial-and-error it is easily available for anyone to use. Trial-and-error was the guiding force in decision making for many participants from the start of the course, and in some cases, to the end of the course. Students may be diverting to trial-and-error heuristic to ease the cognitive load.

Not-knowing can serve as a spark in creating a plan to solve a problem, but if fractured knowledge exists, it is likely that not-knowing will not cause yield an incorrect answer. Just like in the example concerning fractured knowledge, it can be determined that as long as the individual has "fractured knowledge", it will impede the initial awareness of not-knowing.

These themes seem to be closely related to Funke's (2017) theory of unconscious thought. All recurring themes could potentially be unconscious thoughts. Not being aware of such thoughts could prove more difficult for individuals to overcome.

Mason (1999) makes it clear how important not-knowing is as a first step for knowing to occur. This study focused on finding out how students are expressing their not-knowing. Results demonstrated that the participants had difficulty expressing their not-knowing. Four recurring themes came to light from analyzing participants' transcriptions, reflections, and interviews. Examples of deflection, pressure, lack of heuristic

sense, and fractured knowledge were discussed to demonstrate how each affected not-knowing awareness. Understanding these themes can better help in minimizing the problem while maximizing the potential of not-knowing as a lead to knowledge and understanding. The findings might serve as a stepping stone to further research of not-knowing.

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