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Double-layer potential of axially symmetric Helmholtz lowest term equation

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Abstract

1. The classical method for solving boundary value problems is the potential method. Potential kernels are recorded as linear combinations of normal derivatives of the fundamental solutions relating to the corresponding equations. For equations with constant coefficients,the fundamental solutions have,as a rule,a simple form. And as a consequence,it is not difficult to calculate any normal derivatives of them. This topic is described in some university textbooks. Due to numerous applications,the study of equations with singular coefficients is significant in the modern theory of differential equations with partial derivatives. The fundamental solutions are usually recorded as a power series for singular equations; they are limited to the first term in the further studies,and evaluation formulas are used for everything else. Finding a fundamental solution in an explicit form is in itself a significant result. 2. For a singular equation,which is the generalized Helmholtz equation,both with the lowest term and without it,a double-layer potential is found in this work. Potential kernel is a normal derivative of the fundamental solution. In order to find this derivative a special function unit is used. 3. The fundamental solution to the considered equation is expressed in terms of the confluent Horn function. Formula for calculating the normal derivative of this fundamental solution is obtained in the article. The result is written in an explicit form using the same Horn function. Relevance of calculating the normal derivative of the fundamental solution to the axially symmetric Helmholtz equation is determined by its demand in applications and absence of a general theory for this type of equations. 4. When using the formulas associated with the special hypergeometric functions,there was a wide choice to the authors. Using the so-called transformation formula,the result could be recorded in various ways,and not always compactly. The result is recorded in such a way that in calculating the limit values of the potentials the relevant terms will be able to be combined. 5. The potential constructed in this work can be applied to solve any boundary value problems for axially symmetric Helmholtz equation. According to the same scheme,potentials for higher-order equations can be constructed.

Keywords

Axially symmetric Helmholtz equation, Confluent Horn function, Double-layer potential, Fundamental solution, Gaussian hypergeometric function, Normal derivative