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Solution of elliptic optimal control problem with pointwise and non-local state constraints

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Abstract

© 2017, Allerton Press, Inc.We study an optimal control problem of a system governed by a linear elliptic equation, with pointwise control constraints and pointwise and non-local (integral) state constraints. We construct a finite-difference approximation of the problem, we prove the existence and the convergence of the approximate solutions to the exact solution. We construct and study mesh saddle point problem and its iterative solution method and analyze the results of numerical experiments.

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Keywords

constrained saddle point problem, elliptic optimal control, finite difference approximation, iterative methods, state constraint

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