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Modeling of fluid flow in periodic cell with porous cylinder using a boundary element method

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ABSTRACT

The problem of viscous incompressible flow past a periodic array of porous cylinders (a model of flow in an aerosol filter) is solved. The approximate periodic cell model of Kuwabara is used to formulate the fluid flow problem. The Stokes flow model is then adopted to model the flow outside the cylinder and the Darcy law of drag is applied to find the filtration velocity field inside the porous cylinder. The boundary value problems for biharmonic and Laplace equations for stream functions outside and inside the porous cylinder are solved using a boundary elements method. A good agreement of numerical and analytical models is shown. The analytical formulas for the integrals in the expressions for the stream function, vorticity and Cartesian velocity components are obtained. It is shown that the use of analytical integration gives considerable advantage in computing time.

1. Introduction

The problem of air flow through porous bodies has practical applications for many different environmental problems, for example air cleaning devices – aerosol filters, wire screens, aerosol respirators. It is known that the use of porous cylinders as elements of aerosol filters significantly increases the efficiency of the deposition of aerosol particles [1,2]. Also, during the filtration process the suspended particles, deposited on fibers, form a porous layer. The distortion of the fluid flow through the filter elements covered by the porous layers considerably affects the inertial and diffusional capture of further aerosol particles [3]. Hence it is important that efficient mathematical models of fluid flow past porous bodies are developed in order to calculate the two-phase flows of dusty air.

In the work presented here the fluid flow in a periodic array of porous cylinders, modeling the air flow in an aerosol filter, is considered under the assumption of viscous incompressible flow. The approximate periodic cell model of Kuwabara [4] is used to formulate the fluid flow problem. The cell model with Kuwabara boundary conditions was used previously by Stechkina [5], Kirsh [1] to determine the velocity field of the flow over a porous cylinder in the case of small Reynolds number flow, using analytical solution and the collocation method. In this work the Stokes flow model is adopted outside the cylinder in the cell and inside the porous cylinder the Darcy law of drag is applied to find the filtration velocity field. The resulting boundary value problems for the biharmonic and Laplace equations for the stream function and pressure, with boundary conditions on the porous surface, are formulated. The boundary elements method (BEM) is then used to solve the considered boundary value problems. The analytical formulas for the integrals in the expressions for stream function, vorticity and Cartesian velocity components are found. The numerical solution is compared with analytical solution obtained with Kuwabara boundary conditions.

The problems of fluid flow through the porous cylinder in a rectangular domain and fluid flow between eccentric cylinders were also solved by the method developed in this paper and results obtained.

2. Problem statement

The two-dimensional flow of an incompressible gas in a periodic cell, of radius h, with a porous cylinder of radius R_c at a small Reynolds numbers is considered. The radius R_c and the fluid velocity U, at the cell boundary, are used as length and velocity scales. Due to the fluid flow symmetry we select as a calculation domain the upper part of the periodic cell.

The outer fluid domain Ω^e is the half annulus region formed by the cell boundary $r = h = 1/\sqrt{\alpha}$ ($\alpha = 1 - \varepsilon$, ε – porosity) and the boundary of the cylinder, $r = 1$. The porous zone Ω^i is the inner half-circle of unity radius (Fig. 1). The problem of fluid flow in domain Ω^e, assuming Stokes flow, can be reduced to the solution

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