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An exact analytical solution of an $\mathbb{R}$-linear conjugation problem for a n-phased concentric circular heterogeneous structure

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ABSTRACT

A 2-D potential steady-state field in an infinite homogeneous matrix with one $(n-1)$-phased concentric annular inclusion is investigated. It is supposed that the power field in this structure is generated by an arbitrary multipole at infinity. The corresponding boundary value problem is reduced to an equivalent functional equation, which is explicitly solved. Effective resistivities of inclusion and energy dissipation into it are analytically evaluated. Equipotential lines and streamlines are presented.

1. Introduction

In this paper we study a disturbance of a given *a'priori* polynomial complex potential by inserting of a $(n-1)$-phased circular inclusion into an infinite planar homogeneous matrix. The inclusion is divided by $n-2$ concentric circles onto $n-1$ isotropic components of distinct resistivities/conductivities.

Parallel and circular layered composites attract significant interest of scientists as well as practitioners for two reasons. From one side, such heterogeneous structures occur very often in nature and technology (see for example momographs [1–4] and articles [5–8]. From another side, corresponding mathematical problems in hydrodynamics, electro- and magnetodynamics are, from our point of view, the easiest ones for analytical solution by the methods of complex analysis. In this connection, perhaps surprisingly, there are currently no closed-form analytical solutions for this class of problems for the most general situation with an arbitrary number of layers and a set of multipoles generating corresponding field. Only particular cases for $n=2$ (the Milne–Thomson theorem [9], p.157, [10,11]) and $n=3$ [12,13] have been properly mathematically investigated. The case of arbitrary n and a single dipole at infinity was considered in the paper [14], where a disturbed complex potential was sought in the form $w_k(z)=c_kz-d_k/z,\ k=\overline{1,n},$ with unknown coefficients. For these coefficients some rather complicated recursive relations were derived that did not yield to get explicit expressions for c_k, d_k.

In our research we concentrate on the case of arbitrary n, when an undisturbed complex potential has only one multipole, situated at infinity. Our aim is to get a closed-form analytical expression for the disturbed complex potential, more

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