Transport in Porous Media 2017 vol.116 N1, pages 115-142

Free Surface Flow in a Microfluidic Corner and in an Unconfined Aquifer with Accretion: The Signorini and Saint-Venant Analytical Techniques Revisited

Kacimov A., Maklakov D., Kayumov I., Al-Futaisi A.

Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia

Abstract

© 2016, Springer Science+Business Media Dordrecht.Steady, laminar, fully developed flows of a Newtonian fluid driven by a constant pressure gradient in (1) a curvilinear constant cross section triangle bounded by two straight no-slip segments and a circular meniscus and (2) a wedge bounded by two rays and an adjacent film bulging near the corner are studied analytically by the theory of holomorphic functions and numerically by finite elements. The analytical solution of the first problem is obtained by reducing the Poisson equation for the longitudinal flow velocity to the Laplace equation, conformal mapping of the corresponding transformed physical domain onto an auxiliary half-plane and solving there the Signorini mixed boundary value problem (BVP). The numerical solution is obtained by meshing the circular sector and solving a system of linear equations ensuing from the Poisson equation. Comparisons are made with known solutions for flows in a rectangular conduit, circular annulus and Philip's circular duct with a no-shear sector. Problem (2) is treated by the Saint-Venant semi-inverse method: the free surface (quasi-meniscus) is reconstructed by a one-parametric family, which specifies a holomorphic function of the first derivative of the physical coordinate with respect to an auxiliary variable. The latter maps the flow domain onto a quarter of a unit disc where a mixed BVP for a characteristic function is solved by the Zhukovsky–Chaplygin method. Velocity distributions in a cross section perpendicular to the flow direction are obtained. It is shown that the change of the type of the boundary condition from no slip to perfect slip (along the meniscus) causes a dramatic increase of the total flow rate (conductance). For example, the classical Saint-Venant formulae for a sector, with all three boundaries being no-slip segments, predict up to four times smaller rate as compared to a free surface meniscus. Mathematically equivalent problems of unconfined flows in aquifers recharged by a constant-intensity infiltration are also addressed.

http://dx.doi.org/10.1007/s11242-016-0767-y

Keywords

Meniscus, Poisson equation, Signorini formula, Viscous film, Zhukovsky–Chaplygin method, Zunker's pendular water slug

References

[1] Abramowitz, M., Stegun, I.A.: Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. Dover, New York (1965)

[2] Adler, P.M.: Porous Media: Geometry and Transports. Butterworth/Heinemann, Boston (1992)

[3] Ajaev, V.S.: Interfacial Fluid Mechanics. A Mathematical Modeling Approach. Springer, New York (2012)

[4] Al-Futaisi, A., Patzek, T.W.: Impact of wettability on two-phase flow characteristics of sedimentary rocks: quasi-static model. Water Resour. Res. 39(2), 1042–1055 (2003a)

[5] Al-Futaisi, A., Patzek, T.W.: Three-phase hydraulic conductances in angular capillaries. Soci. Petro. Engrg. J. 8(3), 252–261 (2003b)

[6] Al-Maktoumi, A., Kacimov, A., Al-Ismaily, S., Al-Busaidi, H.: Infiltration into two-layered soil: the Green-Ampt and Averyanov models revisited. Transp. Porous Media 109, 169–193 (2015). doi:10.1007/s11242-015-0507-8

[7] Arutyunyan, NKh, Abramyan, B.L.: Torsion of Elastic Bodies. Fizmatgiz, Moscow (in Russian) (1963)

[8] Averyanov, S.F.: The dependence of permeability of soils on their air content. Dokl. AN SSSR 69, 141–144 (1949)

[9] Avkhadiev, F.G., Kacimov, A.R.: Analytical solutions and estimates for microlevel flows. J. Porous Media 8, 125–148 (2005)

[10] Babskii, V.G., Kopachevskii, N.D., Myshkis, A.D., Slobozhanin, L.A., Tyuptsov, A.D.: Fluid Mechanics of Weightlessness. Nauka, Moscow (in Russian) (1976)

[11] Baret, J.-C., Decre, M.M.J., Herminghaus, S., Seemann, R.: Transport dynamics in open microfluidic grooves. Langmuir 23, 5200–5204 (2007)

[12] Berhanu, M., Petroff, A., Devauchelle, O., Kudrolli, A., Rothman, D.H.: Shape and dynamics of seepage erosion in a horizontal granular bed. Phys. Rev. E 86, 041304 (2012)

[13] Blossey, R.: Thin Liquid Films. Dewetting and Polymer Flow. Springer, Dordrecht (2012)

[14] Blunt, M.: Flow in porous media—pore-network models and multiphase flow. Curr. Opin. Colloid Interface Sci. 6, 197–207 (2001)

[15] Blunt, M.J., Jackson, M.D., Piri, M., Valvatne, P.H.: Detailed physics, predictive capabilities and macroscopic consequences for pore-network models of multiphase flow. Adv. Water Resour. 25, 1069–1089 (2002)

[16] Brauner, N., Rovinsky, J., Maron, D.M.: Analytical solution for laminar–laminar two-phase stratified flow in circular conduits. Chem. Eng. Commun. 141–142, 103–143 (1996)

[17] Brinkmann, M., Khare, K., Seemann, R.: Control of liquids by surface energies. In: Hardt, S., Schönfeld, F. (eds.) Microfluidic Technologies for Miniaturized Analysis Systems, pp. 157–197. Berlin (2007)

[18] Constantinescu, V.N.: Laminar Viscous Flow. Springer, Berlin (1995)

[19] Darhuber, A.A., Troian, S.M., Reisner, W.W.: Dynamics of capillary spreading along hydrophilic microstripes. Phys. Rev., E 64(3), 031603 (2001)

[20] Dullien, F.A.L.: Porous Media: Fluid Transport and Pore Structure, 2nd edn. Academic Press, New York (1992)

[21] Finn, R.: Equilibrium Capillary Surfaces. Springer, Berlin (1986)

[22] Gakhov, F. D.: Boundary Value Problems. Nauka, Moscow (in Russian). (English translation of the 1st edn., Addison Wesley, New York, 1966) (1997)

[23] Goldstein, A., Ullmann, A., Brauner, N.: Characteristics of stratified laminar flows in inclined pipes. Int. J. Multiphase Flow 75, 267–287 (2015)

[24] Gurevich, M.I.: Theory of Jets in Ideal Fluids. Academic Press, New York (1965)

[25] Hammecker, C., Barbiero, L., Boivin, P., Maeght, J.L., Diaw, E.H.B.: A geometrical pore model for estimating the microscopical pore geometry of soil with infiltration measurements. Transp. Porous Media 54, 193–219 (2004)

[26] Happel, J., Brenner, H.: Low Reynolds Number Hydrodynamics. Prentice Hall, Englewood Cliffs (1965)

[27] Held, R.J., Celia, M.A.: Modeling support of functional relationships between capillary pressure, saturation, interfacial area and common lines. Adv. Water Resour. 24, 325–343 (2001)

[28] Herminghaus, S., Brinkmann, M., Seemann, R.: Wetting and dewetting of complex surface geometries. Ann. Rev. Mater. Res. 38, 101–121 (2008)

[29] Hui, M.H., Blunt, M.J.: Effects of wettability on three-phase flow in porous media. J. Phys. Chem. B 104, 3833–3845 (2000)

[30] Ilyinsky, N.B., Kacimov, A.R.: The estimation of integral seepage characteristics of hydraulic structures in terms of the theory of inverse boundary-value problems. Z. Angew. Math. Mech., B 72(2), 103–112 (1992)

[31] Kachinsky, N.A.: Soil Physics. V.II. Moscow, Vyshsaya Shkola (in Russian) (1970)

[32] Kacimov, A.R.: Optimization of the protrusion shape for a Couette type flow. Optim. Control Appl. Methods 15, 193–203 (1994)

[33] Kacimov, A.R., Kayumov, I.R.: Viscous flow through straight pore channels. J. Porous Media 3, 199–208 (2002)

[34] Kacimov, A.R., Kayumov, I.R., Al-Maktoumi, A.: Rainfall induced groundwater mound in wedge-shaped promontories: the Strack–Chernyshov model revisited. Adv. Water Resour. 97, 110–119 (2016). http://www.sciencedirect.com/science/article/pii/S0309170816303633

[35] Kacimov, A.R., Obnosov, YuV, Al-Maktoumi, A., Al-Balushi, M.: How much of floating LNAPL can a phreatic surface sustain? Riesenkampf's scheme revisited. Water Resour. Res. 47, W11521 (2011). doi:10.1029/2010WR010369

[36] Kacimov, A.R., Obnosov, Yu., Mosavat, N.: Analytical solution for supercritical upconing of two immiscible fluids moving to a horizontal well. J Pet Sci Eng, submitted

[37] Khare, K., Herminghaus, S., Baret, J.C., Law, B.M., Brinkmann, M., Seemann, R.: Switching liquid morphologies on linear grooves. Langmuir 23, 12997–13006 (2007)

[38] Khare, K., Zhou, J., Yang, S.: Tunable open channel microfluidics on soft poly(dimethylsiloxane) (PDMS) substrates with sinusoidal grooves. Langmuir 25, 12794–12799 (2009)

[39] Kim, S., Karrila, S.J.: Microhydrodynamics: Principles and Selected Applications. Dover, New York (2005)

[40] Kitron-Belinkov, M., Marmur, A., Trabold, T., Dadheech, G.V.: Groovy-drops: effect of groove curvature on spontaneous capillary flow. Langmuir 23, 8406–8410 (2007)

[41] Kołodziej, J.A., Fraska, A.: Elastic torsion of bars possessing regular polygon in cross-section using BCM. Comput. Struct. 84, 78–91 (2005)

[42] Lazouskaya, V., Jin, Y., Or, D.: Interfacial interactions and colloid retention under steady flows in a capillary channel. J. Colloid Interface Sci. 303, 171–184 (2006)

[43] Mahdavi, A., Seyyedian, H.: Steady-state groundwater recharge in trapezoidal-shaped aquifers: a semi-analytical approach based on variational calculus. J. Hydrol. 512, 457–462 (2014)

[44] Mathworks: Partial Differential Equation Toolbox Users Guide. The Mathworks Inc, Natick (1998)

[45] McComb, H.G.: Torsional stiffness of thin-walled shells having reinforcing cores and rectangular, triangular, or diamond cross section. NACA Technical Report 1359, Langley Aeronautical Lab (1957)

[46] Moffatt, H.K., Duffy, B.R.: Local similarity solutions and their limitations. J. Fluid Mech. 96, 299–313 (1980)

[47] Nardin, C.L., Weislogel, M.M.: Capillary driven flows along differentially wetted interior corners. NASA Report, CR - 2005-213799, 1–24 (2005)

[48] Or, D., Tuller, M.: Hydraulic conductivity of partially saturated fractured porous media: flow in a cross-section. Adv. Water Resour. 26(1), 883–898 (2003)

[49] Patzek, T.W., Kristensen, J.D.: Shape factor and hydraulic conductance in noncircular capillaries: II. Two-phase creeping flow. J. Colloid Interface Sci. 236(2), 305–317 (2001)

[50] Patzek, T.W., Silin, D.B.: Shape factor and hydraulic conductance in noncircular capillaries: I. One-phase creeping flow. J. Colloid Interface Sci. 236(2), 295–304 (2001)

[51] Petroff, A.P., Devauchelle, O., Abrams, D.M., Lobkovsky, A.E., Kudrolli, A., Rothman, D.H.: Geometry of valley growth. J. Fluid Mech. 673, 245–254 (2011)

[52] Petroff, A.P., Devauchelle, O., Seybold, H., Rothman, D.H.: Bifurcation dynamics of natural drainage networks. Phil. Trans. R. Soc., A 371, 20120365 (2013)

[53] Philip, J.R.: Flow in porous media. Annu. Rev. Fluid Mech. 2, 177–204 (1970)

[54] Philip, J.R.: Flows satisfying mixed no-slip and no-shear conditions. J. Appl. Math. Phys. (ZAMP) 23, 353–372 (1972)

[55] Polubarinova-Kochina, P.Ya.: Theory of Ground-Water Movement. Moscow Nauka, in Russian (English translation of the first edition: Princeton Univ. Press, Princeton, 1962) (1977)

[56] Polzin, K.A., Choueiri, E.A.: Similarity parameter for capillary flows. J. Phys. D Appl. Phys. 36, 3156–3167 (2003)

[57] Pozrikidis, C.: Introduction to Theoretical and Computational Fluid Dynamics. Oxford University Press, Oxford (2011)

[58] Quere, D.: Wetting and roughness. Annu. Rev. Mater. Res. 38, 71–99 (2008)

[59] Ransohoff, T.C., Radke, C.J.: Laminar flow of a wetting liquid along the corners of a predominantly gas-occupied noncircular pore. J. Colloid Interface Sci. 121, 392–401 (1988)

[60] Rejmer, K., Dietrich, S., Napiórkowski, M.: Filling transition for a wedge. Phys. Rev. E 60, 4027–4042 (1999)

[61] Roth-Nebelsick, A., Ebner, M., Miranda, T., Gottschalk, V., Voigt, D., Gorb, S., Stegmaier, T., Sarsour, J., Linke, M., Konrad, W.: Leaf surface structures enable the endemic Namib desert grass Stipagrostis sabulicola to irrigate itself with fog water. J. R. Soc. Interface (2012). doi:10.1098/rsif.2011.0847

[62] Rye, R.R., Mann Jr., J.A., Yost, F.G.: The flow of liquids in surface grooves. Langmuir 12, 555–565 (1996)

[63] Saint-Venant, B.: Sur la torsion des prismes à bases mixtiligne, et sur une singularité que peuvent offrir certains emplois de la coordonnée logarithmique du système cylindrique isotherme de Lamé. Comptes Rendus des Séances de l'Académie des Sciences. Paris 87, 849–854 (1878). (in French)

[64] Seemann, R., Brinkmann, M., Herminghaus, S., Khare, K., Law, B.M., McBride, S., Kostourou, K., Gurevich, E., Bommer, S., Herrmann, C., Michler, D.: Wetting morphologies and their transitions in grooved substrates. J. Phys. Condens. Matter. 23, 184108 (2011)

[65] Sekulic, D.P.: Wetting and spreading of liquid metals through open microgrooves and surface alterations. Heat Transf. Eng. 32(7–8), 648–657 (2011). doi:10.1080/01457632.2010.509758

[66] Selvadurai, A.P.S.: Partial Differential Equations in Mechanics 2. The Biharmonic Equation, Poisson's Equation. Springer (2000)

[67] Shahraeeni, E., Or, D.: Pore-scale analysis of evaporation and condensation dynamics in porous media. Langmuir 26(17), 13924–13936 (2010)

[68] Slezkin, N.A.: Dynamics of Viscous Incompressible Fluid. Gostechizdat, Moscow (in Russian) (1955)

[69] Strack, O.D.L.: Groundwater Mechanics. Prentice-Hall Inc, Englewood Cliffs (1989)

[70] Teo, C.J., Khoo, B.C.: Analysis of Stokes flow in microchannels with superhydrophobic surfaces containing a periodic array of micro-grooves. Microfluidics Nanofluidics 7, 353–382 (2009)

[71] Timoshenko, S.P., Goodier, J.C.: Theory of Elasticity. McGraw-Hill, New York (1970)

[72] Uflyand, Y.S.: Integral Transforms in Problems of Elasticity Theory. Nauka, Leningrad (in Russian) (1968)

[73] Versluys, T.: Die Kapillaritat der Boden. Internet. Mitt, fur Bodenkunde, Bd. 7, Berlin (in German) (1917)

[74] Wang, C.Y.: Torsion of a compound bar bounded by cylindrical polar coordinates. Q. J. Mech. Appl. Math. 48, 359–400 (1995)

[75] Wang, C.Y.: Torsion of polygonal bar with core of different material. J. Engrg. Mech. 125, 1218–1221 (1999)

[76] Wang, C.Y.: Flow over a surface with parallel grooves. Phys. Fluids 15, 1114–1121 (2003)

[77] Weislogel, M.M.: Compound capillary rise. J. Fluid Mech. 709, 622–647 (2012)

[78] White, F.M.: Viscous Fluid Flow. McGraw-Hill, New York (1991)

[79] Wigglesworth, L.A., Stevenson, A.C.: Flexure and torsion of cylinders with cross-sections bounded by orthogonal circular arcs. Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, 170, 391–414 (1939)

[80] Wolfram, S.: Mathematica: a system for doing mathematics by computer. Addison-Wesley, Redwood City (1991)

[81] Youngs, E.G., Kacimov, A.R.: Conduction through spherical particles at low liquid content. Int. J. Heat Mass Transf. 50(1–2), 292–302 (2007)

[82] Zhang, Q., Karadimitriou, N.K., Hassanizadeh, S.M., Kleingeld, P.J., Imhof, A.: Study of colloids transport during two-phase flow using a novel polydimethylsiloxane micro-model. J. Colloid Interface Sci. 401, 141–147 (2013)

[83] Zhou, D., Blunt, M.J., Orr, F.M.: Hydrocarbon drainage along corners of noncircular capillaries. J. Colloid Interface Sci. 187, 11–21 (1997)

[84] Zunker, F.: Das Verhalten des Bodens zum Wasser. Handbuch der Bodenlehre. Bd. VI, Berlin (in German) (1930)