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## Toward homological structure theory of semimodules: On semirings all of whose cyclic semimodules are projective

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## ABSTRACT

In this paper, we introduce homological structure theory of semirings and CP-semirings — semirings all of whose cyclic semimodules are projective. We completely describe semi-simple, Gelfand, subtractive, and anti-bounded, CP-semirings. We give complete characterizations of congruence-simple subtractive CP-semirings and congruence-simple anti-bounded semirings, which solve two earlier open problems for these classes of semirings. We also study in detail the properties of semimodules over Boolean algebras whose endomorphism semirings are CP-semirings; and, as a consequence of this result, we give a complete description of ideal-simple CP-semirings.

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