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## **Eigenvibrations of a bar with elastically attached load**

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## Abstract

© 2017, Pleiades Publishing, Ltd.We study the problem on the eigenvibrations of a bar with an elastically attached load. The problem is reduced to finding the eigenvalues and eigenfunctions of an ordinary secondorder differential problem with a spectral parameter nonlinearly occurring in the boundary condition at the load attachment point. We prove the existence of countably many simple positive eigenvalues of the differential problem. The problem is approximated by a grid scheme of the finite element method. We study the convergence and accuracy of the approximate solutions.

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