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On idempotent τ-measurable operators affiliated to a von Neumann algebra

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Abstract

Let τ be a faithful normal semifinite trace on a von Neumann algebra M, let p, 0 < p < ∞, be a number, and let Lp(M, τ) be the space of operators whose pth power is integrable (with respect to τ). Let P and Q be τ-measurable idempotents, and let A ≡ P − Q. In this case, 1) if A ≥ 0, then A is a projection and QA = AQ = 0; 2) if P is quasinormal, then P is a projection; 3) if Q ∈ M and A ∈ Lp(M, τ), then A2 ∈ Lp(M, τ). Let n be a positive integer, n > 2, and A = An ∈ M. In this case, 1) if A ≠ 0, then the values of the nonincreasing rearrangement μt(A) belong to the set {0} ∪ [‖An−2‖−1, ‖A‖] for all t > 0; 2) either μt(A) ≥ 1 for all t > 0 or there is a t0 > 0 such that μt(A) = 0 for all t > t0. For every τ-measurable idempotent Q, there is aunique rank projection P ∈ M with QP = P, PQ = Q, and PM = QM. There is a unique decomposition Q = P + Z, where Z2 = 0, ZP = 0, and PZ = Z. Here, if Q ∈ Lp(M, τ), then P is integrable, and τ(Q) = τ(P) for p = 1. If A ∈ L1(M, τ) and if A = A3 and A − A2 ∈ M, then τ(A) ∈ R.

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Keywords

Hilbert space, idempotent, integrable operator, non-increasing rearrangement, normal trace, projection, quasinormal operator, rank projection, von Neumann algebra, τ-compact operator, τ-measurable operator