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Electroweak parameters of the Z^0 resonance and the standard model

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The four LEP experiments have each performed precision measurements of Z^0 parameters. A method is described for combining the results of the four experiments, which takes into account the experimental and theoretical systematic errors and their correlations. We apply this method to the 1989 and 1990 LEP data, corresponding to approximately 650 000 Z^0 decays into hadrons and charged leptons, to obtain precision values for the Z^0 parameters. We use these results to test the standard model and to constrain its parameters.

1. Introduction

The four LEP experiments are providing measurements of unprecedented precision of the Z^0 resonance and decay parameters. These measurements permit a better understanding of both the strong and electroweak interactions. For the best possible confrontation of these experimental results [1–4] with the theory, we present here combined results of the four Collaborations for the parameters derived from the hadronic and leptonic line shapes as well as the leptonic forward–backward asymmetry.

Each of the four LEP experiments fits cross sections and asymmetries to extract electroweak parameters. The combination of the four results raises some technical questions on the correct treatment of the differences in the analysis methods of the four experiments, as well as the proper treatment of common experimental and theoretical errors. These issues are discussed in section 2. The combined results are given in section 3. The implications for the standard model of electroweak and strong interactions are discussed in section 4.

2. How the results are combined

The programs used by the LEP experiments to parametrize the line-shape and lepton forward–backward asymmetries have been described in detail elsewhere [5,6]. To compare the procedures used by each experiment, a common data set was fitted using the procedures developed by each experiment. Differences among the results were small compared to the sensitivity of the total LEP data.

The following systematic uncertainties are correlated among the experiments:
– LEP absolute energy scale [7]: 20 MeV; this uncertainty affects only M_Z.
– LEP point-to-point energy uncertainty: 10 MeV, assumed uncorrelated for each energy point; this uncertainty corresponds to a systematic error of 5 MeV on Γ_Z.
– Luminosity theoretical uncertainty: the estimated luminosity uncertainty for each experiment ranged from 0.3% to 0.5%. We assigned 0.3% as an estimate of the correlated part of the uncertainty.
– Bhabha t-channel parametrization: all four experiments used the same Bhabha parametrization program [8]. Each experiment assumed an uncertainty of 0.5% of the total t-channel contribution which we assume to be fully correlated among them.

Other potential sources of correlated uncertainties among the four experiments, including those due to the use of common Monte Carlo programs for acceptance corrections, were estimated to be negligible. The effect of the correlated uncertainties on the lepton forward–backward asymmetry measurements was negligible for the current data sets.

To combine the results of the four experiments, we have checked that it is sufficient to perform a weighted average of the fit parameters directly. This

[1] Lists of authors can be found in refs. [1–4].

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method is correct if the errors are sufficiently gaussian, and if the common systematic errors are treated properly. These errors where subtracted quadratically before averaging the results, and were then included in the final error. The advantage of this procedure over combining cross section and asymmetry data is that of simplicity. The procedure has been checked by comparing the averaged results to those derived from combined fits to the measured cross sections and asymmetries of the four experiments. The maximum deviation for any parameter between the two sets of results was less than one tenth of the uncertainty in that parameter, and is therefore negligible. Studies have shown that interpretation of the fit results is not sensitive to details of the correlation matrix used, and that the average correlation matrix adequately describes the combined parameters. The results given in the following section were obtained by averaging the four sets of resonance parameters and using the average correlation matrix.

3. Results

The four experiments express the Z resonance measurements in terms of different sets of parameters. Measured values for a set of these parameters which has been published explicitly by the four experiments [1,4] are summarized in table 1. The average correlation matrix of the four experiments for these parameters is given in table 2.

For the mass and width of the Z^0, the values given correspond to the definition based on the Breit–Wigner denominator $s-M_Z^2+is\Gamma_Z/M_Z$. Throughout this paper parameters are given after deconvolution of initial state radiation and represent only the contribution of Z^0 exchange.

Table 1
Line shape and lepton coupling constants from the four LEP experiments as published in refs. [1–4]. The common systematic errors are 20 MeV for M_Z, 5 MeV for Γ_Z, 0.25% for Γ_e, Γ_μ, Γ_τ, Γ_ℓ and $g_{A\ell}^2$, and 0.3% for σ_h^0. The 5 MeV of common error on Γ_Z are not included for ALEPH and L3. For other common errors see text.

	ALEPH	DELPHI	L3	OPAL
M_Z (GeV)	91.182 ±0.009	91.177 ±0.010	91.181 ±0.010	91.161 ±0.009
Γ_Z (GeV)	2.485 ±0.017	2.465 ±0.020	2.501 ±0.017	2.492 ±0.016
σ_h^0 (nb)	41.44 ±0.36	41.84 ±0.45	41.1 ±0.4	41.01 ±0.41
Γ_e (MeV)	83.8 ±0.9	82.4 ±1.2	83.3 ±1.1	82.9 ±1.0
Γ_μ (MeV)	81.4 ±1.4	86.9 ±2.1	84.5 ±2.0	83.2 ±1.5
Γ_τ (MeV)	82.4 ±1.6	82.7 ±2.4	84.0 ±2.7	82.7 ±1.9
Γ_ℓ (MeV)	83.1 ±0.7	83.4 ±0.8	83.6 ±0.8	83.0 ±0.7
$g_{V\ell}^2$ [a]	$(1.76 \pm 0.66)\times10^{-3}$	$(0.3 \pm 1.0)\times10^{-3}$	$(2.12 \pm 1.20)\times10^{-3}$	$(0.58 \pm 0.70)\times10^{-3}$
$g_{A\ell}^2$ [a]	0.2480±0.0020	0.2508±0.0027	0.2500±0.0028	0.2495±0.0023

[a] The values quoted in the ALEPH and L3 papers were for v_ℓ and a_ℓ, rather than for $g_{V\ell}^2$ and $g_{A\ell}^2$. ALEPH: $v_\ell=-0.042^{+0.009}_{-0.007}$ and $a_\ell=-0.498\pm0.002$. L3: $v_\ell=-0.046^{+0.015}_{-0.012}$ and $a_\ell=-0.500\pm0.003$. The values quoted in the OPAL paper were in the different notation using $\hat{v}_\ell^2\equiv4g_{V\ell}^2$ and $\hat{a}_\ell^2\equiv4g_{A\ell}^2$ rather than $g_{V\ell}^2$ and $g_{A\ell}^2$: $\hat{v}_\ell^2=0.0023\pm0.0028$ and $\hat{a}_\ell^2=0.998\pm0.009$.

Table 2
Average correlation coefficients for parameter set of table 1. The entries given by a "–" correspond to lepton universality.

	Γ_Z	σ_h^0	Γ_e	Γ_μ	Γ_τ	Γ_ℓ	$g_{V\ell}^2$	$g_{A\ell}^2$
M_Z	0.09	0.01	0.06	0.04	0.03	0.08	0.08	0.02
Γ_Z		−0.25	0.48	0.26	0.22	0.63	0.02	0.59
σ_h^0			0.22	0.14	0.11	0.30	0.00	0.28
Γ_e				−0.19	−0.16	–	–	–
Γ_μ					0.33	–	–	–
Γ_τ						–	–	–
Γ_ℓ							–	–
$g_{V\ell}^2$								−0.32

Averages for the four LEP experiments from Z^0 line shape and charged lepton forward–backward asymmetry have been calculated for the parameters given in table 1, as well as for some other sets of parameters. One such set consists of M_Z, Γ_Z, σ_h^0 the hadronic cross section at $s=M_Z^2$, $R_e \equiv \Gamma_{had}/\Gamma_e$, $R_\mu \equiv \Gamma_{had}/\Gamma_\mu$ and $R_\tau \equiv \Gamma_{had}/\Gamma_\tau$. These parameters relate most directly to the observables measured by the experiments and are the least correlated ones. Alternatively the last three parameters can be replaced by the Z^0 partial widths for decay into hadrons and lepton pairs. The leptonic partial widths in particular are sensitive to the electroweak sector and are essentially free from QCD uncertainties. As a third set of parameters, we give the leptonic branching ratios. All of these parameters are listed in table 3.

The lepton measurements are in agreement with lepton universality. Therefore, the values for $R_\ell \equiv \Gamma_{had}/\Gamma_\ell$, Γ_ℓ, the vector and axial-vector couplings of Z^0 to charged leptons, $g_{V\ell}$, $g_{A\ell}$ and the lepton forward–backward asymmetry at the pole, A_{FB}^0, are determined assuming lepton universality. The leptonic forward–backward asymmetry at the Z^0 pole is given by [#1]

$$A_{FB}^0 = 3\frac{g_{V\ell}^2 g_{A\ell}^2}{(g_{V\ell}^2+g_{A\ell}^2)^2}\,.$$

The results are given in table 4. Additional correlation matrices for the most commonly used sets of parameters are given in tables 5 and 6.

[#1] Effects coming from photon exchange, as well as real and imaginary parts of the photon vacuum polarization are excluded from this definition and accounted for in the fitting formulae used by the experiments.

Table 3
Average LEP line shape parameters. Values for χ^2 of the weighted average are quoted for those parameters given directly by the four experiments.

Parameter	Average value	χ^2
M_Z (GeV)	91.175±0.021	3.4
Γ_Z (GeV)	2.487±0.010	2.0
σ_h^0 (nb)	41.33 ±0.23	2.3
R_e	20.91 ±0.22	
R_μ	20.88 ±0.18	
R_τ	21.02 ±0.23	
Γ_e (MeV)	83.20 ±0.55	1.0
Γ_μ (MeV)	83.35 ±0.86	5.1
Γ_τ (MeV)	82.76 ±1.02	0.3
Br($Z^0 \to e^+e^-$) (%)	3.345±0.020	
Br($Z^0 \to \mu^+\mu^-$) (%)	3.351±0.034	
Br($Z^0 \to \tau^+\tau^-$) (%)	3.328±0.040	

Table 4
Average LEP Z^0 parameters assuming lepton universality. Values for χ^2 of the weighted average are quoted for those parameters given directly by the four experiments.

Parameter	Average value	χ^2
R_ℓ	20.89 ±0.13	
Γ_{had} (GeV)	1.740 ±0.012	
Γ_ℓ (MeV)	83.24 ±0.42	0.4
Br($Z^0 \to$ hadrons) (%)	69.93 ±0.31	
Br($Z^0 \to \ell^+\ell^-$) (%)	3.347 ±0.013	
A_{FB}^0	0.0138±0.0049	
$g_{V\ell}^2$	(1.16 ±0.41)×10^{-3}	2.9
$g_{A\ell}^2$	0.2493±0.0013	0.8
$g_{V\ell}^2/g_{A\ell}^2$	0.0047±0.0017	
Γ_{inv} (MeV)	498 ±8	
Γ_{inv}/Γ_ℓ	5.985 ±0.095	

Table 5
Correlation coefficients for Z resonance parameters.

	Γ_Z	σ_h^0	R_e	R_μ	R_τ	R_ℓ	A_{FB}^0
M_Z	0.09	0.01	0.00	0.00	0.00	0.00	0.08
Γ_Z		−0.25	−0.07	−0.03	−0.03	−0.07	0.01
σ_h^0			0.10	0.13	0.10	0.18	0.00
R_e				0.07	0.05	–	–
R_μ					0.07	–	–
R_τ						–	–
R_ℓ							0.02

Table 6
Correlation coefficients for Z resonance parameters. The values in column 5 are calculated without assuming lepton universality. With universality the correlation between Γ_Z and Γ_{had} is 0.61.

	Γ_Z	Γ_e	Γ_μ	Γ_τ	Γ_{had}	Γ_ℓ
M_Z	0.09	0.06	0.04	0.03	0.05	0.08
Γ_Z		0.48	0.26	0.22	0.47	0.63
Γ_e			−0.19	−0.16	−0.31	–
Γ_μ				0.33	0.58	–
Γ_τ					0.49	–
Γ_{had}						0.26

The Z^0 partial width to invisible final states, Γ_{inv}, is defined as $\Gamma_{\text{inv}} = \Gamma_Z - \Gamma_{\text{had}} - 3\Gamma_\ell$ while the ratio $\Gamma_{\text{inv}}/\Gamma_\ell$ is extracted from the data using

$$\Gamma_{\text{inv}}/\Gamma_\ell = \sqrt{\frac{12\pi R_\ell}{M_Z^2 \sigma_h^0}} - R_\ell - 3 .$$

Unlike Γ_{inv} itself, this ratio depends only on the hadronic peak cross section and $\Gamma_{\text{had}}/\Gamma_\ell$ and is essentially independent of Γ_Z. The values obtained for these two quantities are listed in table 4.

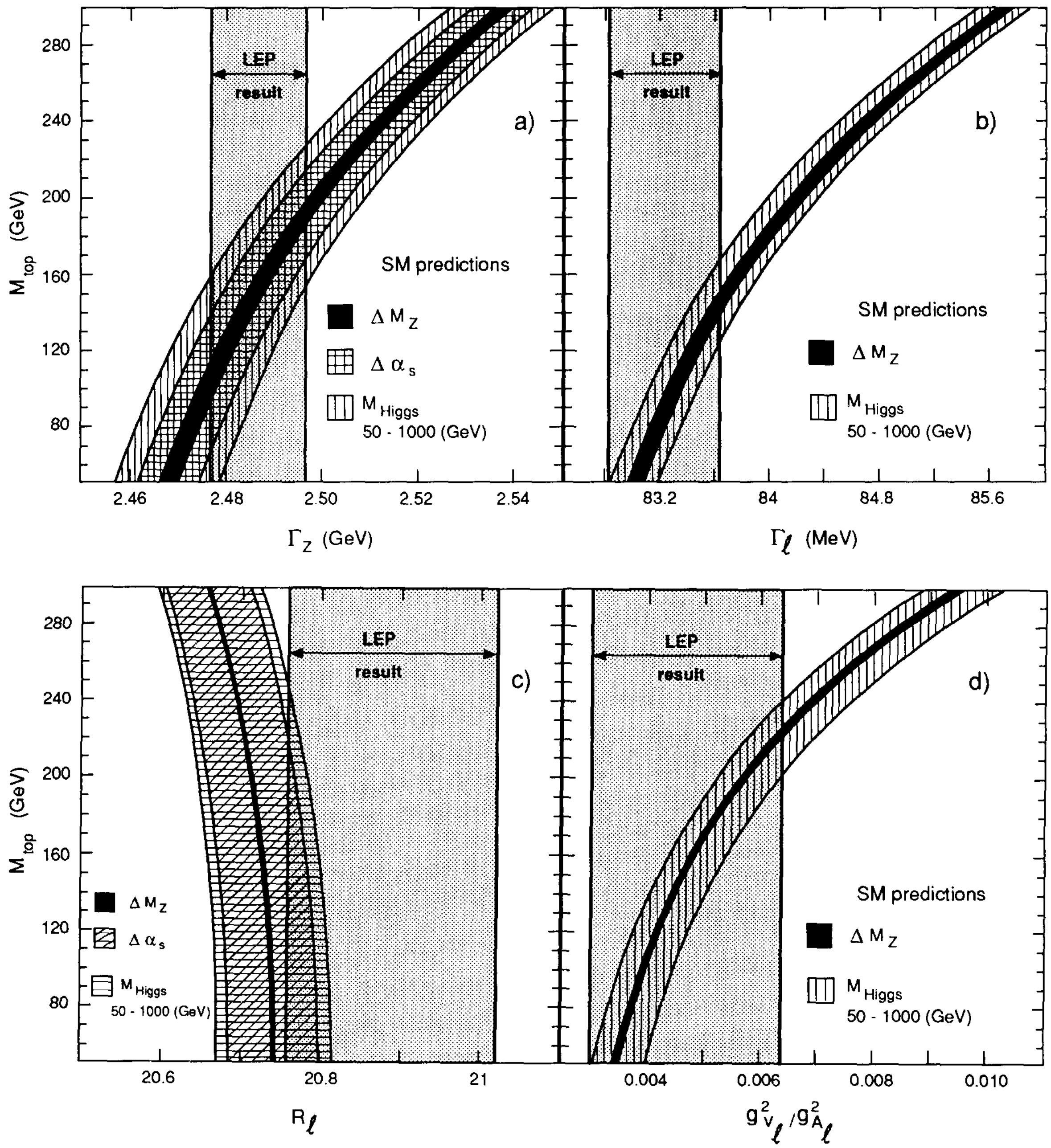

Fig. 1. Z parameters as a function of M_t together with standard model prediction. (a) Γ_Z, (b) Γ_ℓ, (c) R_ℓ, (d) $g_{V\ell}^2/g_{A\ell}^2$.

4. The LEP results and the standard model

The impact of the above results on the standard model can be stated as follows:

- They give more accurate values for some of the parameters of the model.
- They provide strong evidence that the standard model contains the three known fermion families and no others.
- They provide checks on the consistency of the model which are an order of magnitude more sensitive than previous experiments.

The parameters of the model which are either directly measured or deduced from these results are the Z^0 mass, the weak mixing angle, the top-quark mass and the strong coupling constant. The latter three will be discussed below. The value for the Z^0 mass, $M_Z=91.175\pm0.021$ GeV was given in section 3.

In the standard model, each light neutrino species (light here means much lighter than $\frac{1}{2}M_Z$) contributes Γ_ν to the invisible width of the Z^0. The ratio of the partial widths to neutrinos and charged leptons in the standard model is given by

$$\Gamma_\nu/\Gamma_\ell=1.993\pm0.004\ .$$

This ratio is particularly insensitive to radiative corrections, as shown by its very small uncertainty. This error was estimated by allowing the top mass to vary from 90 to 200 GeV.

Thus the measurement of $\Gamma_{\text{inv}}/\Gamma_\ell$ can be used to determine

$$N_\nu=3.00\pm0.05\ .$$

The quoted error includes 0.02 coming from the common theoretical systematic uncertainty in the luminosity determination.

Assuming that all neutrinos have masses much smaller than $\frac{1}{2}M_Z$, this result excludes a fourth fermion family. Moreover, the fact that N_ν within its small error, is consistent with an integer, constitutes a sensitive test of the electroweak theory.

For a given value of M_Z the standard model predictions for the quantities Γ_Z, Γ_ℓ, as well as the asymmetry A^0_{FB} can be calculated. They are sensitive to M_t and M_H, through radiative corrections [5,6]. The top-quark mass dependence is dominated by an M_t^2 term in the region of interest, and is large compared to the sensitivity of the measurements. The additional Higgs mass effect is logarithmic and small, so that at present useful conclusions about the Higgs mass are not possible. In fig. 1 we show the measurements of Γ_Z, Γ_ℓ, R_ℓ and $g^2_{V\ell}/g^2_{A\ell}$ together with their standard model dependence on M_t. In these calculations we have constrained the strong coupling constant to be $\alpha_s=0.118\pm0.008$. [9]. We have assumed $M_H=300$ GeV and have included a variation of M_H in the range 50–1000 GeV in the errors.

In fig. 2 standard model constraints for Z line-shape parameters and forward–backward asymmetries are shown as function of the effective weak mixing angle, $\sin^2\theta^{\text{lept}}_{\text{eff}}$, and M_t. In addition, we show constraints coming from measurements of M_W and M_W/M_Z from CDF [10] and UA2 [11], and from measurements of the neutrino neutral to charged current ratios from CDHS [12] and CHARM [13]. Here, $\sin^2\theta^{\text{lept}}_{\text{eff}}$ is defined in terms of the ratio of the leptonic vector and axial-vector coupling strengths

$$\sin^2\theta^{\text{lept}}_{\text{eff}}=\tfrac{1}{4}(1-g_{V\ell}/g_{A\ell})\ .$$

With a fit to Γ_Z, σ^0_h, $g^2_{V\ell}$ and $g^2_{A\ell}$ we have determined M_t and the strong coupling constant α_s simultaneously. We have used the central value $M_H=300$ GeV and the constraint $M_Z=91.175\pm0.021$ GeV. In addition we have performed this fit including the

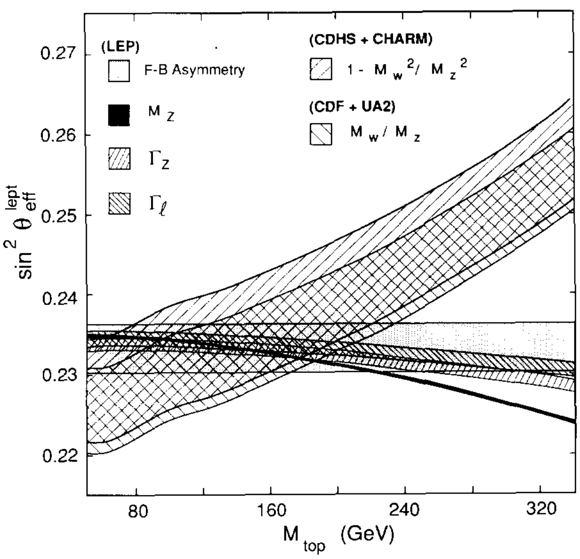

Fig. 2. Constraints on $\sin^2\theta^{\text{lept}}_{\text{eff}}$ versus M_t from different measurements corresponding to 1σ limits.

Table 7
Results of fits to LEP and other data for M_t and α_s. In the first and third column α_s is unconstrained whereas, in the others it is constrained to the value 0.118 ± 0.008. In the third and fourth column data are included from the $p\bar{p}$ experiments UA2 [1]: $M_W/M_Z = 0.8813 \pm 0.0041$, and CDF [10]: $M_W = 79.91 \pm 0.39$ GeV and from the neutrino experiments, CDHS [12] and CHARM [13]: $\sin^2\theta_w = 0.2300 \pm 0.0064$. The first error is experimental while the second error corresponds to 50 GeV $< M_H <$ 1000 GeV. The dots for the negative error of M_t indicate that it reaches the bound for open top production, which is not implemented in the parametrizations used.

	α_s unconstrained	α_s constrained	α_s unconstrained + collider and ν data	α_s constrained + collider and ν data
M_t (GeV)	$94^{+53+23}_{-\ldots-24}$	124^{+40+21}_{-56-21}	124^{+28+16}_{-31-18}	132^{+27+18}_{-31-19}
α_s	$0.141 \pm 0.017 \pm 0.002$	0.123 ± 0.007	0.138 ± 0.015	0.123 ± 0.007
χ^2/d.o.f.	0.3/2	2.2/3	1.5/5	3.0/6
$\sin^2\theta_{eff}^{lept}$	–	$0.2337 \pm 0.0014^{+0.0001}_{-0.0004}$	$0.2337 \pm 0.0010^{+0.0003}_{-0.0004}$	$0.2335 \pm 0.0009^{+0.0001}_{-0.0004}$
$\sin^2\theta_w \equiv 1 - M_W^2/M_Z^2$	–	$0.2299^{+0.0067}_{-0.0048}$	0.2299 ± 0.0033	0.2290 ± 0.0033
M_W (GeV)	–	$80.01^{+0.27}_{-0.37}$	80.01 ± 0.19	80.06 ± 0.19

constraint $\alpha_s = 0.118 \pm 0.008$. We have also repeated both fits using M_W and M_W/M_Z measurements from $p\bar{p}$ and neutrino experiments. The results are summarized in table 7. A variation of M_H in the range 50–1000 GeV contributes to the systematic error of M_t while it is negligible for α_s. Also shown in table 7 are the corresponding values for $\sin^2\theta_{eff}^{lept}$, $\sin^2\theta_w \equiv 1 - M_W^2/M_Z^2$ and M_W. The quality of the fits is illustrated in figs. 3a and 3b where the χ^2 of the fit is shown as a function of M_t.

5. Conclusions

Combining these measurements of the four LEP experiments results in a considerably enhanced sensitivity with respect to that afforded by any one of the four data sets. Directly combining the parameters derived by the LEP experiments reproduces well the results of simultaneous fits to the four sets of cross-section and lepton forward–backward asymmetries.

The Z line shape parameters and the lepton for-

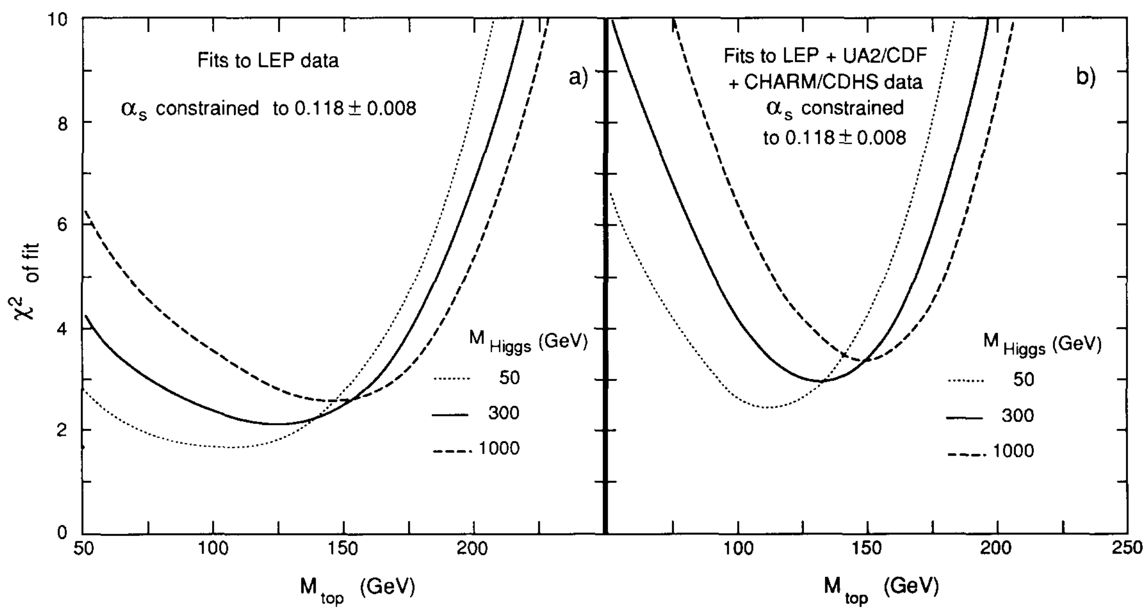

Fig. 3. χ^2 as function of M_t from a fit to (a) LEP data with three degrees of freedom, (b) LEP data and $p\bar{p}$ collider and ν data for five degrees of freedom.

ward–backward asymmetry have been determined with a new level of precision. These results check the consistency of the standard model with an order of magnitude greater sensitivity than previous experiments. With the expected reduction in the statistical and systematic errors of the LEP experiments, more stringent checks will be possible in the future.

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