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High order approximation for the coverage probability by a confident set centered at the positive-part James–Stein estimator

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ABSTRACT

In this paper we continue our investigation connected with the new approach developed in Ahmed et al. [Ahmed, S.E., Saleh, A.K.Md.E., Volodin, A., Volodin, I., 2006. Asymptotic expansion of the coverage probability of James–Stein estimators. Theory Probab. Appl. 51 (4) 1–14] for asymptotic expansion construction of coverage probabilities, for confidence sets centered at James–Stein and positive-part James–Stein estimators. The coverage probabilities for these confidence sets depend on the noncentrality parameter τ^2, the same as the risks of these estimators. In this paper we consider only the confidence set centered at the positive-part James–Stein estimator. As is shown in the above-mentioned reference, the new approach provides a method to obtain for the given confidence set, an asymptotic expansion of the coverage probability as one formula for both cases $\tau \to 0$ and $\tau \to \infty$. We obtain the third terms of the asymptotic expansion for both mentioned cases, that is, the coefficients at τ^2 and τ^{-2}. Numerical illustrations show that the third term has only a small influence on the accuracy of the asymptotic estimation of coverage probability.

1. Introduction

The problem of confidence estimation of the mean vector $\theta = (\theta_1, \ldots, \theta_p)$ for the p-dimensional normal distribution with independent components and equal variances $\sigma^2 = 1$ is considered. Let $\bar{X} = (\bar{X}_1, \ldots, \bar{X}_p)$ be the vector of sample means calculated by samples of common size n from the marginal distributions. The confidence set

$$D_{\bar{X}} = \left\{\theta : n\sum_{1}^{p}(\theta_i - \bar{X}_i)^2 \le c^2\right\}$$

has the given confidence coefficient $1-\alpha$, if c^2 is the quantile of chi-square distribution with p degrees of freedom given by the relation $K_p(c^2) = 1-\alpha$, where $K_p(\cdot)$ is the chi-square distribution function.

This confidence set possesses the minimax property, but there exist other minimax sets that obtain bigger coverage probability for all values of the noncentrality parameter $\tau^2 = n\|\theta\|^2$ if $p \ge 4$. In this paper we consider one of these sets

$$D_{\delta^+} = \{\theta : n\|\theta - \delta^+(\bar{X})\|^2 \le c^2\},$$

which is centered at the positive-part James and Stein (1961) estimator given by

$$\delta^+(\bar{X}) = \left(1 - \frac{p-2}{n\|\bar{X}\|^2}\right)\bar{X}\mathbf{I}\{n\|\bar{X}\|^2 > p-2\}$$

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