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Singularities of meniscus at the V-shaped edge

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ABSTRACT

An understanding of the capillary rise of the meniscus formed on the V-shaped fibers is crucial for many applications. We classified the cases when the meniscus cannot be smooth by analyzing the local behavior of the solutions to the Laplace equation of capillarity near the sharp edge. The V-angle and two contact angles that the meniscus forms on two chemically different sides of the fiber form a 3D phase space. Smooth menisci constitute a special domain in this 3D space. The constructed diagram allows one to separate the solutions with smooth and non-smooth menisci. The obtained criteria were illustrated using chemically inhomogeneous plates, blades, square corners, and Janus V-shaped edges.

In the Jurin capillary rise experiment, a fiber is immersed perpendicularly to the free liquid surface [1]. The air/liquid interface deforms into a meniscus. Depending on the fiber wettability, the contact line, separating wet and dry parts of the fiber surface, can bend into different configurations. The shapes of the contact line and meniscus are very sensitive to the surface properties of a fiber [2–6]. In many cases, classification of the substrate wettability is associated with the behavior of the contact line [6]. In design of omniphobic materials, one needs to understand the effect of the fiber shape on the formation of singularities on the contact line [7,8]. When the fiber surface is not smooth and have some V-shaped grooves, Concus and Finn [9] showed that the nonlinear Laplace equation of capillarity may have a family of solutions with the infinite first derivative at the wetting boundary. It is therefore instructive to analyze the behavior of meniscus at the fiber surface having not only some V-shaped grooves, but also the V-shaped sharp edges. An asymptotic solution of the Laplace equation of capillarity near the sharp V-type edges revealed the criteria for the development of non-smooth menisci.

In the Cartesian system of coordinates (x, y, z), the meniscus profile $z = h(x, y)$ describes the liquid elevation above the reference plane (x, y), which coincides with the horizontal liquid level far away from the fiber. The center of coordinates is chosen at the V-edge O shown in Fig. 1. The Laplace law of capillarity [10], $\sigma(1/R_1 + 1/R_2) - \rho gH = 0$ is employed to describe the meniscus shape where the first term in this equation is the mean curvature defined by two principle radii of curvatures R_1 and R_2; the second term is the hydrostatic pressure, σ is the surface tension of the liquid, ρ is its density, and g is acceleration due to gravity. The Young–Laplace equation is used to formulate the boundary condition at the surfaces of each face of the wedge with different contact angles, γ^+ and γ^- [6,10,11]. It is convenient to rewrite the mean curvature in terms of the outward normal vector $\mathbf{N}$ to the meniscus surface, $(1/R_1 + 1/R_2) = -\nabla\cdot\mathbf{N}$, where this vector is expressed through the surface elevation as $\mathbf{N} = (1 + |\nabla h|^2)^{-1/2}(-\partial h/\partial x, -\partial h/\partial y, 1)$ [12]. Thus, the mathematical model is written as [13,14]

$$\Omega:\quad \nabla\cdot\left[(1 + |\nabla h|^2)^{-1/2}\nabla h\right] - \frac{\rho g h}{\sigma} = 0, \tag{1.1}$$

$$\Gamma^{\pm}:\quad (1 + |\nabla h|^2)^{-1/2}\frac{\partial h}{\partial n} = -\cos\gamma^{\pm}, \tag{1.2}$$

where Ω is the domain occupied by the liquid, and $\mathbf{n}$ is the outward unit normal vector to the boundaries $\Gamma^{\pm}$ (Fig. 1a).

Since we are interested only in the behavior of meniscus in the vicinity of the sharp edge, the boundary condition at infinity is relaxed. First we need to define what we mean under the smooth and singular menisci. Writing Eq. (1.1) in the form $-\nabla\cdot\mathbf{N} = \rho g h/\sigma$ one immediately infers that the normal vector $\mathbf{N}$ must be continuous and differentiable in Ω, hence the meniscus should be smooth in Ω. Singularities are permissible only at the boundary $\Gamma = \Gamma^+ \cup \Gamma^-$. These singularities can be generated, for example, by the singularities of the boundary.

Consider for a moment the general case of a complex-shaped fiber. To accommodate the effect of the fiber shape in the definition of the smooth meniscus, we will call the meniscus smooth when its normal vector $\mathbf{N}$ changes continuously along the contact line

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