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The projective geometric theory of systems of second-order differential equations: Straightening and symmetry theorems

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Abstract

In the framework of the projective geometric theory of systems of differential equations, which is being developed by the authors, conditions which ensure that a family of graphs of solutions of a system of m second-order ordinary differential equations $\ddot{y\rightarrow} = f\rightarrow(t, y\rightarrow, \dot{y\rightarrow})$ with m unknown functions y1(t), ⋯ , ym(t) can be straightened (that is, transformed into a family of straight lines) by means of a local diffeomorphism of the variables of the system which takes it to the form $z\rightarrow'' 00 = 0$ (straightens the system) are investigated. It is shown that the system to be straightened must be cubic with respect to the derivatives of the unknown functions. Necessary and sufficient conditions for straightening the system are found, which have the form of differential equations for the coefficients of the system or are stated in terms of symmetries of the system. For m = 1 the system consists of a single equation $\ddot{y} = f\rightarrow(t, y, \dot{y})$, and the tests obtained reduce to the conditions for straightening this equations which were derived by Lie in 1883. © 2010 RAS(DoM) and LMS.

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Keywords

Associated projective connection, Projective geometric theory of systems of second-order ordinary differential equations, Straightening theorems, Symmetry group