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Log-periodic corrections to the Cole–Cole expression in dielectric relaxation

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ABSTRACT

A model of the self-similar process of relaxation is given, and a method of derivation of the kinetic equations for the total polarization based on the ideas of fractional kinetics is suggested. The derived kinetic equations contain integro-differential operators having non-integer order. They lead to the Cole–Cole expression for the complex dielectric permittivity. It is shown rigorously that the power-law exponent α in the Cole–Cole expression coincides with the dimension of the mixed space-temporal fractal ensemble. If the discrete scale invariance for the temporal-space structure of the dielectric medium considered becomes important, then the expression for the complex dielectric permittivity contains log-periodic corrections (oscillations) and, hence, it generalizes the conventional Cole–Cole expression. The corrections obtained in this model suggest another way of interpretation and analysis of dielectric spectra for different complex materials.

1. Introduction

At the present time, detailed research in soft condensed physics matter [1] as a means of understanding the structure dynamics and relaxation phenomena in complex systems is observed. These complex systems represent a wide class of different materials with disordered structure. In particular, the investigation of (bio)polymers, colloids (emulsions and microemulsions), biologic cells, porous materials and liquid crystals can be also related to a complex system. In most methods used to investigate disordered materials, dielectric spectroscopy plays a dominant role on the mesoscopic scale [1]. However, the simple exponential relaxation law and classical model of Brownian diffusion cannot describe a wide class of relaxation phenomena and kinetics in these soft condensed materials. So, it is necessary to develop other approaches for the description of nonexponential relaxation behavior and anomalous diffusion processes (which nowadays are referred to as "strange kinetics" [2]).

Usually, for detailed descriptions of the kinetic processes taking place in different complex systems, it is necessary to use many different experimental methods to cover the corresponding frequency range. From this point of view, dielectric spectroscopy has an undoubted advantage over all other methods, because modern dielectric spectrometers allow one to cover a very wide frequency range (from 10^{-6} to 10^{11} Hz) [3–5].

The total polarization $P(t)$ of a dielectric placed in an external electric field $E(t)$ contains two parts [6]:

$$P(t) = P_0(t) + P_1(t), \tag{1}$$

where $P_0(t)$ defines the instantaneous component while $P_1(t)$ determines its retarded component. In the classic theory of relaxation [6], it is supposed that the rate of change of the retarded component is proportional to the difference between its

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