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An equilibrium thermostatistics of a nonextensive finite system: Canonical distribution and entropy

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ABSTRACT

A simple model is presented to illustrate the equilibrium thermostatistics of a nonentensive finite system. Interaction between the finite system and the reservoir is taken into account as a nonextensive term $\lambda H_1 H_2$ in the expression of total energy (H_1 and H_2 are the energy of the finite system and the reservoir respectively, λ is nonadditivity parameter). In the present paper, a case with harmonic reservoir potential is considered. Energy probability distribution, average energy, heat capacity and entropy function for energy distribution are derived in different finite systems including those with constant density of state in energy, the ideal gas and the phonon gas.

A system can be called finite size or a small system when its size is comparable with the interaction scale between its elements. A finite system is a family of complex systems to which the paradigms of statistical physics for large system are challenged. One challenge is that the hypothesis of extensive energy and entropy may become invalid for systems in which the interacting scale is comparative to its size or the surface effect is not negligible with respect to the volume effect. Another challenge may be related to the fluctuation of many thermodynamic quantities such as temperature, energy, pressure etc.. A possible consequence of that is the failure of the statistics theory based on the exponential probability distribution. Hence, more and more interest is transferred to the nonextensive system (see for example Ref. [1]). We also notice many pros and cons around nonextensive statistical theory derived from the Tsallis entropy with the help of the maximum entropy principle (maxent) [2].

Finding the probability distribution of energy is one of the most important things to work out for the statistical description of a thermodynamic system. The difficulty for a finite system is that we know nothing about the entropy and the energy nonextensivity in general. In other words, the knowledge is generally missing about the entropy function of probability and about the mutual dependence in energy of interacting parts. Hence, for example, if we look at a closed system in contact with a reservoir (canonical ensemble), we cannot even write the relationship between total energy H of the union system + reservoir and H_1 and H_2, respectively the energy of the system and the reservoir. Worse, the thermodynamic limit of large number of constituents does not exist. All these hypotheses we had for a large extensive system are missing now. It becomes impossible to derive a probability distribution with the mathematical tricks in textbooks (e.g., Stirling formula, $H_1 \ll H_2$ or $H = H_1 + H_2$ approximation, maxent, etc.). Nevertheless, there are many efforts to derive the probability distribution of energy from the first principle (assuming equiprobable microstates of isolated system) [3–7]. The work of Ref. [7] is the first

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