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Locally most powerful sequential tests of a simple hypothesis vs. one-sided alternatives

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ABSTRACT

Let $X_1, X_2, \ldots$ be a discrete-time stochastic process with a distribution P_θ, $\theta \in \Theta$, where Θ is an open subset of the real line. We consider the problem of testing a simple hypothesis H_0: $\theta = \theta_0$ vs. a composite alternative H_1: $\theta > \theta_0$, where $\theta_0 \in \Theta$ is some fixed point. The main goal of this article is to characterize the structure of locally most powerful sequential tests in this problem.

For any sequential test (ψ, ϕ) with a (randomized) stopping rule ψ and a (randomized) decision rule ϕ let $\alpha(\psi, \phi)$ be the type I error probability, $\dot{\beta}_0(\psi, \phi)$ the derivative, at $\theta = \theta_0$, of the power function, and $\mathcal{N}(\psi)$ an average sample number of the test (ψ, ϕ). Then we are concerned with the problem of maximizing $\dot{\beta}_0(\psi, \phi)$ in the class of all sequential tests such that

$$\alpha(\psi, \phi) \le \alpha \quad \text{and} \quad \mathcal{N}(\psi) \le \mathcal{N},$$

where $\alpha \in [0,1]$ and $\mathcal{N} \ge 1$ are some restrictions. It is supposed that $\mathcal{N}(\psi)$ is calculated under some fixed (not necessarily coinciding with one of P_θ) distribution of the process $X_1, X_2, \ldots$.

The structure of optimal sequential tests is characterized.

1. Introduction

Let $X_1, X_2, \ldots, X_n, \ldots$ be a discrete-time stochastic process with a distribution P_θ, $\theta \in \Theta$, where Θ is an open subset of the real line. We consider the problem of testing a simple hypothesis $H_0 : \theta = \theta_0$ vs. a composite alternative $H_1 : \theta > \theta_0$, where $\theta_0 \in \Theta$ is some fixed point. The main goal of this article is to characterize the structure of locally most powerful, in the sense of Berk (1975), sequential tests in this problem.

We follow Novikov (2009b) in the definitions and notation related to sequential hypothesis tests, as well as their interpretation and characteristics (see also Wald, 1950; Ferguson, 1967; DeGroot, 1970; Schmitz, 1993; Ghosh et al., 1997, among many others).

In particular, we say that a pair (ψ, ϕ) is a sequential hypothesis test if

$$\psi = (\psi_1, \psi_2, \ldots, \psi_n, \ldots) \quad \text{and} \quad \phi = (\phi_1, \phi_2, \ldots, \phi_n, \ldots),$$

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