Applied Mathematical Modelling 39 (2015) 3380–3397

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Groundwater flow in hillslopes: Analytical solutions by the theory of holomorphic functions and hydraulic theory

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ARTICLE INFO

Article history:
Received 26 May 2012
Received in revised form 9 May 2014
Accepted 3 November 2014
Available online 4 December 2014

Keywords:
Potential theory
Hodograph
Hydraulic approximation
Unconfined seepage over corner
Three-component heterogeneity
Hydrogeology of alluvium aquifers in Oman

ABSTRACT

Three 2-D steady Darcian flows in an aquifer with a subjacent confining layer of a non-constant slope or a bedding inconformity are studied by two models: a potential theory (conformal mappings, the inverse boundary-value problem method, and the theory of R-linear conjugation) and hydraulic approximation. First, flow over a corner, whose vertex is either a stagnation point or point of infinite Darcian velocity, is analysed as a transition from one "normal" regime upstream to another downstream. The hodograph domain is a circular triangle, which is mapped onto a complex potential strip via an auxiliary half-plane. Parametric equations (backwater curves) for the phreatic surface are obtained. For the same flow problem, a depth-averaged 1-D nonlinear ODE for the thickness of the saturated zone (a generalized Dupuit–Fawer model) is numerically solved showing a perfect match with the potential (2-D) solution. Second, a non-planar aquifuge boundary is reconstructed as a streamline, along which an additional "control" boundary condition holds in the form of pore pressure as a function of an auxiliary variable (a relation between the hydraulic head and vertical Cartesian coordinate). The free surface is found in terms of Cauchy's integrals for the Zhukovskii function, with explicit integrations for selected "controls". Third, a confined flow in a two-layered aquifer having a lens-type semi-circular inclusion in the subjacent stratum and incident velocity parallel to the interface between two aquifers is examined. The conjugation conditions along all four boundaries, across which the hydraulic conductivity jumps, are exactly met. The three velocity fields are explicitly presented, with examination of the flow net, including separatrices ("capture zone" boundaries), demarcating suction/barriering of the lens, and evaluation of the lens-induced cross-flow (commingling) between the two strata.

1. Introduction and hydrogeological motivation

There is a constant yearning for all that is unconfined.
F. Holderlin *Mnemosyne*.

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http://dx.doi.org/10.1016/j.apm.2014.11.016
0307-904X/