Applied Mathematical Modelling 33 (2009) 1970–1981

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A generalized Milne–Thomson theorem for the case of parabolic inclusion

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ARTICLE INFO

Article history:
Received 27 June 2007
Received in revised form 4 April 2008
Accepted 1 May 2008
Available online 10 May 2008

Keywords:
Heterogeneous media
Circular theorem
Analytic functions

ABSTRACT

Complex analysis methods are applied to determine a velocity field of seepage in a heterogeneous infinite planar medium consisting of two dissimilar homogeneous components with a parabolic interface. New cases with arbitrary singularities of the principal part of a required complex potential are considered.

1. Introduction

We consider the problem of determination of a seepage field in a planar porous medium with a parabolic division boundary between two homogeneous components having the hydraulic conductivity k_1 and k_2. The interface separates the inclusion S_2 from the ambient medium S_1 as is shown in Fig. 1. We assume a steady-state, fully-saturated, Darcian flow in a layer (e.g., a confined aquifer) of a constant thickness. Then the Darcian velocity $\mathbf{v}_p(x,y) = (v_{px}, v_{py})$, $(x,y) \in S_p$, is potential and solenoidal in each isotropic component S_p, $p = 1, 2$ [1]:

$$\operatorname{div} \mathbf{v}_p(x,y) = 0, \quad \operatorname{rot} \mathbf{v}_p(x,y) = 0. \tag{1.1}$$

Along the parabolic interface $\mathscr{L} = \bar{S}_1 \cap \bar{S}_2$ the limit values of the vectors $\mathbf{v}_p$ and $\mathbf{v}_p/k_p$ satisfy the following refraction conditions:

$$[\mathbf{v}_1(x,y)]_n = [\mathbf{v}_2(x,y)]_n, \quad k_2[\mathbf{v}_1(x,y)]_\tau = k_1[\mathbf{v}_2(x,y)]_\tau, \quad (x,y) \in \mathscr{L}. \tag{1.2}$$

Here the subscripts n and τ denote the normal and tangential components of the vectors. Problem (1.1), (1.2) can be equivalently formulated in terms of the complex potential $\mathbf{w}_p(x,y) = (\varphi_p(x,y), \psi_p(x,y))$, where $\varphi_p(x,y)$ is the velocity potential and $\psi_p(x,y)$ is the stream function within the component S_p, and $\partial\varphi_p/\partial x = v_{px}$, $\partial\varphi_p/\partial y = v_{py}$.

It is well–known that an explicit analytical solution of the above stated problem can be obtained for few heterogeneities only. If we exclude the trivial case of perfectly aligned strata [1], then the simplest two-component composite consists of a circular inclusion. Solution of the corresponding boundary value problem constitutes the famous Milne–Thomson circle theorem [2]. In [2]. In [3,24] the Milne–Thomson circle theorem was generalized for the case when a required complex potential had a finite number of singularities arbitrary situated on the plane. These singularities physically represent pumping and/or injection wells (sinks/sources, [4,5]), river-locks or dams (vortexes, [6]) and immersed obstacles (dipoles, [7]).

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doi:10.1016/j.apm.2008.05.004