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Constructal design of permeable reactive barriers: Groundwater-hydraulics criteria

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Abstract

Unidirectional, steady-state, Darcian flow in a confined homogeneous aquifer is partially intercepted by a permeable reactive barrier (PRB), the shape of which is optimized with the following hydraulic criteria: seepage flow rate through a PRB (equivalent to the width and frontal area of the intercepted part of the plume in 2-D and 3-D cases, correspondingly) and travel time of a marked particle through the PRB interior along streamlines. The wetted perimeter, cross-sectional area and volume of the reactive material are selected as isoperimetric constraints. The PRB contour is modeled as either a constant head line (if the reactive material is much more permeable than the aquifer) or as a refraction boundary (if the reactive material has an arbitrary permeability), on which the hydraulic head and normal flux components in the barrier and aquifer are continuous. In the former case, the complex potential domain of the flow is a tetragon and a broad class of PRBs can be studied. In the latter case, analytical solutions are available for ellipses and ellipsoids (only these classes of shapes are considered in optimization). In the 2-D case and constant head PRB, a novel shape-control technique through the kernels of singular integrals is implemented: the Zhukovskii function is introduced; a Dirichlet boundary-value problem is solved for this function by setting the orientation (with respect to the incident flow direction) of the Darcian velocity vector on the PRB contour as a control function. Unlike similar controls for impermeable airfoils in aerodynamic design, the kernel has two discontinuities, which reflect the flow topology near a hinge (stagnation) point and the PRB tip. The integral is evaluated for V-shaped and curve-shaped PRBs and parametric expressions for the contours are obtained resulting (for the latter case) in a "pointy banana" shape. In the class of a V-shaped PRB, it is proved that a straight-line barrier minimizes the perimeter if the plume width is fixed. In 2- and 3-D refracting PRBs, the Pilatovskii (ellipse) and Poisson (ellipsoid) solutions for the flow field inside and outside the PRB are used for obtaining explicit formulae for the magnitude of the velocity, which is uniform inside the PRB. Simple expressions for the longest travel time within the PRB and the discharge intercepted by it are obtained. The ellipse/ellipsoid axes ratio/ratios are used as control variables in optimization. Extrema are obtained and analyzed for different PRB-aquifer conductivity ratios and for varying angles between the incident velocity vector and the ellipse/ellipsoid axes. © 2011 Springer Science+Business Media B.V.

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Keywords

Analytical solution, Capture zone, Darcian velocity, Optimal design, Permeable reactive barrier,

Travel time