Logic, Methodology and Philosophy of Science IX
D. Prawitz, B. Skyrms and D. Westerståhl (Editors)

CONTRIBUTIONS TO THE HISTORY OF VARIATIONS OF WEAK DENSITY IN THE n-R.E. DEGREES

MARAT M. ARSLANOV

Department of Mathematics, University of Kazan

A set $A \subseteq \omega$ is called n-r.e. if $n = 1$ and A is recursively enumerable (r.e) or $n > 1$ and there is an r.e. set A_1 and an n-r.e. set $A_2 \subseteq A_1$ such that $A = A_1 - A_2$. A Turing degree is called a n-r.e. degree if it contains a n-r.e. set; it is called properly n-r.e. if it is n-r.e. but not $n-1$-r.e. Clearly a set A is n-r.e. if and only if there is a recursive function f such that for all $x \lim_s f(s,x) = A(x), f(0,x) = 0$ and

(1) $$\text{card}\{s : f(s+1,x) \neq f(s,x)\} \leq n.$$

In the obvious way, a set $A \subseteq \omega$ is called ω-r.e. iff it satisfies the same definition where (1) is replaced by

(1′) $$\text{card}\{s : f(s+1,x) \neq f(s,x)\} \leq h(x)$$

for some recursive h. The reader should note that if a set A satisfies (1′) for some recursive h then it satisfies (1′) for any recursive unbounded function g (see [1]).

The existence of properly α-r.e. degrees was first proved for $1 < \alpha < \omega$ by Cooper [2] and for $\alpha = \omega$ by Epstein [5] and Lachlan (1968, unpublished), who showed that there is an ω-r.e. minimal degree, and that every nonrecursive n-r.e. degree for $1 < n < \omega$ bounds a nonrecursive r.e. degree, respectively.

During the past decade, an intensive study of the structure of n-r.e. (and more particularly d-r.e.=2-r.e.) degrees was initiated. Interest in the n-r.e. degrees stems from their affinity with the r.e. degrees, although a number of recent papers have sought several essential differences between these structures. Probably the most fundamental result in this direction is the Cooper-Harrington-Lachlan-Lempp-Soare Nondensity Theorem, which states that the partial orderings of n-r.e. degrees for any $n > 1$ are not dense.