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## Hysteretic effects in the problems of artificial freezing

Alimov M., Kornev K., Mukhamadullina G.

*Kazan Federal University, 420008, Kremlevskaya 18, Kazan, Russia*

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### Abstract

The technique of the Riemann problem with displacement is used to solve the steady two-dimensional problem of freezing of groundwater flow by two 'freeze pipes'. A series of steady-state limiting configurations is analyzed, and it is shown that as the area of frozen soil around the pipes grows, the two frozen regions eventually 'link' to form a single frozen zone. If the process is reversed (the united body is unfrozen in stages) the 'division' (i.e., unlinking) follows a different path. Hence there is hysteresis in the linking and division of the frozen zone. Mathematically, it appears that within some range of input parameters the problem has several solutions. We obtain the 'phase diagram' for frozen domains and show that the linking condition differs from the division condition by a perceptible amount. This allows the possibility of optimizing the technological regimes of freezing. Namely, once the overlapping of ice columns has occurred, the body could maintain this linked state even if the temperature of the freeze pipes was significantly reduced.

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