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An application of the Ryll-Nardzewski–Woyczyński theorem to a uniform weak law for tail series of weighted sums of random elements in Banach spaces

Tien-Chung Hu^a, Eunwoo Nam^b, Andrew Rosalsky^{c,*}, Andrei I. Volodin^d^aDepartment of Mathematics, Tsing Hua University, Hsinchu, Taiwan 30043, ROC^bDepartment of Computer Science and Statistics, Korean Air Force Academy, Cheongjoo, South Korea 363-849^cDepartment of Statistics, University of Florida, 206 Griffin-Floyd Hall, Box 118545, Gainesville, FL 32611-8545, USA^dResearch Institute of Mathematics and Mechanics, Kazan State University, Kazan 420008, Russia

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Abstract

For a sequence of Banach space valued random elements $\{V_n, n \geq 1\}$ (which are not necessarily independent) with the series $\sum_{n=1}^{\infty} V_n$ converging unconditionally in probability and an infinite array $a = \{a_{ni}, i \geq n, n \geq 1\}$ of constants, conditions are given under which (i) for all $n \geq 1$, the sequence of weighted sums $\sum_{i=n}^m a_{ni} V_i$ converges in probability to a random element $T_n(a)$ as $m \rightarrow \infty$, and (ii) $T_n(a) \xrightarrow{P} 0$ uniformly in a as $n \rightarrow \infty$ where a is in a suitably restricted class of infinite arrays. The key tool used in the proof is a theorem of Ryll-Nardzewski and Woyczyński (1975, Proc. Amer. Math. Soc. 53, 96–98). © 2000 Elsevier Science B.V. All rights reserved

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1. Introduction

In this paper, a uniform weak law of large numbers (WLLN) is established for tail series of weighted sums of Banach space valued random elements. Throughout, $\{V_n, n \geq 1\}$ is a sequence of random elements defined on a probability space (Ω, \mathcal{F}, P) and assuming values in a real separable Banach space \mathcal{X} with norm $\|\cdot\|$. In general, it is not assumed that the $\{V_n, n \geq 1\}$ are independent.

Let $\Theta = \{\theta = (\theta_1, \theta_2, \dots), \theta_n = \pm 1, n \geq 1\}$. The series $\sum_{n=1}^{\infty} V_n$ is assumed to *converge unconditionally in probability*; that is, for every $\theta \in \Theta$, the sequence of partial sums $S_n^\theta \equiv \sum_{i=1}^n \theta_i V_i$ converges in probability to

* Corresponding author. Tel.: +1-352-392-1941 ext: 225; fax: +1-352-392-5175.

E-mail address: rosalsky@stat.ufl.edu (A. Rosalsky)