Stability analysis of traveling wave solution for gravity-driven flow

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A linear stability analysis was performed for three models of flow in unsaturated porous media to determine the conditions for growth of small perturbations. The models considered include the conventional Richards equation (RE), a sharp front Richards equation (SFRE) and an extended Richards equation (RRE). The first two models are based on the use of an equilibrium capillary pressure-saturation function, while the third model is derived using a dynamic capillary pressure-saturation function represented by a relaxation coefficient. A traveling wave solution was formulated for each of the governing equations and used as the basic solution of each model. The stability analysis was based on imposing a small perturbation to the basic solution. The RE model yields only the well-known monotonically decreasing saturation profile toward the wetting front, and the wetting front is unconditionally stable. The SFRE model by its nature has a monotonically increasing saturation profile toward the front and an abrupt drop to the initial saturation. This flow is unconditionally unstable. The RRE model is distinct from the others in that it is the only model that is able to produce truly non-monotonic saturation profiles. The wetting front for the RRE model is conditionally stable, *i.e.* stable for high frequency perturbations, and unstable otherwise. This leads to the existence of a wave-number for maximum amplification, which should relate to the dimensions of fingers in unstable flow.

1. INTRODUCTION

The phenomenon of gravity-driven unstable flow has attracted much interest during the last three decades. Many mathematical models have been developed to attempt to model this phenomenon [1–4]. To describe the fingering, the mathematical model to be developed must bear at least two principal features: (i) the model must be able to generate initial unstable growth of small perturbations, and (ii) it must be able to promote persistence of the initially growing perturbations by limiting lateral spreading behind the unstable front. The experimental results presented by Glass et al. [3] and the physically based theory described by Glass et al. [3] and Nieber [4] demonstrate that the second of these features, finger persistence, is dominated by hysteresis in the capillary pressure-saturation

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