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Stability of an ice sheet on an elastic bed

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Abstract

The stability of stationary flow of a two-dimensional ice sheet is studied when the ice obeys a power flow law (Glen's flow law). The mass accumulation rate at the top is assumed to depend on elevation and span and the bed supporting the ice sheet consists of an elastic layer lying on a rigid surface. The normal perturbation of the free surface of the ice sheet is a singular eigenvalue problem. The singularity of the perturbation at the front of the ice sheet is considered using matched asymptotic expansions, and the eigenvalue problem is seen to reduce to that with fixed ice front. Numerical solution of the perturbation eigenvalue problem shows that the dependence of accumulation rate on elevation permits the existence of unstable solutions when the equilibrium line is higher than the bed at the ice divide. Alternatively, when the equilibrium line is lower than the bed, there are only stable solutions. Softening of the bed, expressed through a decrease of its elastic modulus, has a stabilising effect on the ice sheet.

Keywords: Ice sheet; Stability

1. Introduction

Many geophysical and environmental problems are described by slow spreading of a heavy fluid under the action of gravity [1–3]. The main features of these phenomena are that inertia is negligible and viscous forces balance the gravity-driven pressure gradient force; the aspect ratio of the heavy fluid is small; and the flow direction is close to horizontal [1]. The range of physical phenomena characterised by these features is broad, e.g., the flow of lava [2], lithosphere [4], ice sheets [5] and oil [6]. Here, we focus on ice sheet flow. Ice sheets flow as gravity currents with a non-Newtonian rheology, but are also characterised by regions of mass accumulation and ablation at their upper, free surface. Balance between accumulation upstream and ablation downstream allows the possibility of stationary-state solutions.

In the limit that the accumulation rate does not depend on the elevation, the bed is rigid, and the ice sheet front is fixed, it has been shown that a stationary solution is stable [7–9]. However, the mass accumulation on an ice sheet generally *does* depend on the upper surface elevation [10,11]. This reflects the dependence of weather conditions on elevation above sea level. Typically, in the upper part of an ice sheet, accumulation from snowfall is positive, whilst in the lower part accumulation is negative due to melting and ablation. These regions are separated by the so-called *equilibrium line* of zero accumulation. The dependence of the accumulation rate on elevation can lead to the instability of a stationary ice sheet to perturbations of its free surface. This is caused by the positive feedback between a local elevation of the free surface and the consequent increase of the mass-accumulation rate on it. Since the increase in accumulation causes the ice sheet to advance through an increase of the

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