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Information and Computation 203 (2005) 145–162
Information and Computation
www.elsevier.com/locate/ic

On the computational power of probabilistic and quantum branching program

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Received 14 February 2004; revised 22 April 2004
Available online 26 September 2005

Abstract

In this paper, we show that one-qubit polynomial time computations are as powerful as NC^1 circuits. More generally, we define syntactic models for quantum and stochastic branching programs of bounded width and prove upper and lower bounds on their power. We show that any NC^1 language can be accepted exactly by a width-2 quantum branching program of polynomial length, in contrast to the classical case where width 5 is necessary unless $NC^1 = ACC$. This separates width-2 quantum programs from width-2 doubly stochastic programs as we show the latter cannot compute the middle bit of multiplication. Finally, we show that bounded-width quantum and stochastic programs can be simulated by classical programs of larger but bounded width, and thus are in NC^1. For read-oncequantum branching programs (QBPs), we give a symmetric Boolean

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[1] Work done in part while visiting Institute of Advanced Study and Max-Planck Institute for Mathematics, supported in part by Russia Fund for Basic Research Grant 03-01-00769.
[2] Supported in part by Russia Fund for Basic Research Grant 03-01-00769.
[3] Supported in part by DFG grants, and IST Grant 14036 (RAND-APX).
[4] Supported by NSF Grants PHY-0200909, CCR-0220070, and EIA-0218563.

doi:10.1016/j.ic.2005.04.003