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## Descent methods for mixed variational inequalities in a Hilbert space

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### Abstract

In this paper, properties of merit functions for mixed variational inequalities involving a differentiable cost mapping and a convex nondifferentiable function are considered under a Hilbert space setting, thus extending the previous results in a finite-dimensional space. Moreover, we consider several descent methods for the above problem under various monotonicity assumptions on the cost mapping.

*Key words:* Mixed variational inequality,  $D$ -gap function, descent methods.

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### 1 Introduction

Let  $U$  be a nonempty closed and convex subset of a real Hilbert space  $H$ . We set  $\langle \cdot, \cdot \rangle$  the inner product on  $H$ , and  $\| \cdot \|$  its norm. Let  $G : H \rightarrow H$  be a continuously differentiable mapping, and  $f : H \rightarrow \mathbb{R}$  a convex and continuous

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