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Elastic solutions for a pressurised tube with given exterior displacements

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Abstract

We find the particular solution of the following problem for an elastic tube: given the displacements at $n+1$ levels of the exterior surface and a constant pressure and the 'sliding condition' on the interior surface of the tube it should be possible to find the displacements at any point of the tube. The solution is reduced to a succession of plane boundary problems for the elliptic differential equations. Each boundary value problem is reduced to a system of linear equations.

The method is programmable and applicable for problems of tubing. It is illustrated by application to a pressurised tube subjected to bending and compression.

Keywords: Displacements; Equilibrium equations; Analytic functions; Polynomials; Boundary conditions; Exterior surface; Elastic tube

1. Introduction

We present the formulation and find the solution of a basic problem in the theory of elasticity for a cylindrical elastic tube with the axis of symmetry $O\tilde{Z}$. The displacements at $n+1$ levels of the exterior surface and the constant pressure on the interior surface of the tube are given. The 'sliding condition' on the interior surface is valid. The coordinates of the desired vector of displacements are supposed to be polynomials with respect to the coordinate $\tilde{z}$. The coefficients of the polynomials are the solutions of plane boundary value problems for the elliptic differential equations. These problems are solved successively via reducing to systems of linear equations.

We give the polynomial form of the desired coordinates and satisfy the equilibrium equations and boundary conditions in Section 2. Section 3 is divided into three subsections where the unknown coefficients are found successively. We find coefficients of the highest power of $\tilde{z}$ via the corresponding differential equations and boundary conditions in Section 3.1. These boundary value problems are reduced to systems of linear equations. The functions found in Section 3.1 are used in Section 3.2 for finding coefficients of $\tilde{z}^{n-1}$ by the same way. We find coefficients of $\tilde{z}^k$, $k < n-1$, for desired polynomials in Section 3.3. The method is illustrated by application to a pressurised tube subjected to bending and compression in Section 4.

The same method was applied in Ref. [1] to a similar problem for a cylindrical solid.

2. Formulation of the problem and analysis

Let Ω be a cylindrical tube in the coordinate space $(x, y, \tilde{z})$ where the element of the cylinder is parallel to the $\tilde{z}$-axis, the exterior radius is 1, the interior radius is $\rho, \tilde{z}_i \le \tilde{z} \le \tilde{z}_f$. The intersections of the exterior surface of Ω with the planes $\tilde{z} = \tilde{z}_j, j = 0, 1, \ldots, n$, are denoted by C_j (Fig. 1). We assume a constant pressure p acting at the interior surface of Ω. We also suppose that the vector of stresses which acts on the plane orthogonal to $O\tilde{Z}$ has the plane component which is orthogonal at the interior surface to the normal to the interior surface—we name this condition the sliding condition.

Let $\vec{a} = (u(x, y, \tilde{z}), v(x, y, \tilde{z}), w(x, y, \tilde{z}))$ be the vector of displacements of the points of Ω. Given the vectors of displacements of the curves $C_j, j = 0, 1, \ldots, n$, it should be possible to find the displacements at any point of Ω.

We suppose that every coordinate function of the desired vector of displacements $\vec{a}$ is a polynomial with respect to $\tilde{z}$,

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