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Solving the two-dimensional CIS problem by a rational algorithm[☆]

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Abstract

The CIS problem is formulated as follows. Let p be a fixed integer, $1 \leq p < n$. For given $n \times n$ complex matrices A and B , can one verify whether A and B have a common invariant subspace of dimension p by a procedure employing a finite number of arithmetical operations? We describe an algorithm solving the CIS problem for $p = 2$. Unlike the algorithm proposed earlier by the second and third authors, the new algorithm does not impose any restrictions on A and B . Moreover, when A and B generate a semisimple algebra, the algorithm is able to solve the CIS problem for any p , $1 < p < n$. © 2000 Published by Elsevier Science Inc. All rights reserved.

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1. Introduction

Let A and B be $n \times n$ complex matrices. The CIS problem (where CIS is an abbreviation for “Common Invariant Subspace”) is formulated as follows.

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