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Inequalities for the Perron root related to Levinger's theorem

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Abstract

For the Perron roots of square nonnegative matrices A, B, and $A + D^{-1}B^{\mathrm{T}}D$, where D is a diagonal matrix with positive diagonal entries, the inequality

$$\rho(A + D^{-1}B^{\mathrm{T}}D) \geqslant \rho(A) + \rho(B)$$

is proved under the assumption that A and B have a common unordered pair of nonorthogonal right and left Perron vectors. The case of equality is analyzed. The above inequality generalizes the inequality $\rho(\alpha A + (1-\alpha)B^{\mathrm{T}}) \geqslant \alpha\rho(A) + (1-\alpha)\rho(B)$, proved under stronger assumptions by Bapat, and implies a generalization of Levinger's theorem on the monotonicity of the Perron root of a weighted arithmetic mean of a nonnegative matrix and its transpose. Also, for the Perron root

$$\rho\left(A^{(\alpha)} \circ (D^{-1}A^{\mathrm{T}}D)^{(c-\alpha)}\right), \quad c \geqslant 1, \ 0 \leqslant \alpha \leqslant c,$$

of a weighted (entrywise) geometric mean of A and $D^{-1}A^{\mathrm{T}}D$, where $A^{(\alpha)} = \left(a_{ij}^{\alpha}\right)$ and "$\circ$" denotes the Hadamard product, the monotonicity property dual to that asserted by generalized Levinger's theorem is established. © 1998 Elsevier Science Inc. All rights reserved.

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