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Conduction through a grooved surface and Sierpinsky fractals

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Abstract

Conduction in a semi-infinite wall with a grooved line of contact between the wall material and convective environment is studied using series expansions. A periodic composition of semicircles is shown to result in a uniform gradient distribution at specific values of the groove radius and the convection heat transfer coefficient. Two fractal parquets exposed to natural thermal gradients are studied by the methods of complex analysis. In double periodic patterns each elementary cell is fractal (Sierpinsky's carpet and Sierpinsky's gasket) in which 'dark' and 'light' phases have arbitrary conductivities. The Maxwell approximation is used to calculate effective characteristics of both fractal structures by 'homogenization' of the environment of an 'inclusion'. Solution of an exact two-dimensional refraction problem within an elementary cell including two components is used for upscaling, i.e. recalculation of effective conductivities and dissipations of subfractals of consequently increasing order.

Keywords: Refraction; Homogenization; Effective conductivity; Fractal; Upscaling

1. Introduction

Problems of heat conduction in domains with interfaces between different phases include two juxtaposing areas: heterogeneous media (spatially-varying conductivity) and non-smooth outer boundaries of domains of homogeneous materials, in particular, developed surfaces. Such extended surfaces allow to enhance the cooling of walls. Since extensive reviews are available on both subjects [1,2], the survey of the literature below is reduced to a minimum.

In what follows, we develop further our analytical approach to study steady, two-dimensional (2D) heat conduction [3,4]. The goals of this note are twofold. First, we show that for a specific value of the heat convection transfer coefficient, the temperature gradient is constant within a wall which surface is extended by a periodic system of semi-circular troughs. Second, we apply the analytical solutions of the R-linear conjugation problem [5] to calculate the effective conductivity and dissipation of two double periodic fractal structures.

2. Uniform heat flux from a periodic system of troughs

Consider a semi-infinite wall of conductivity k whose surface is extended by a system of semi-circular troughs of radius a (Fig. 1).

Steady state temperature $T(x, y)$ within the wall

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