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The bases of weighted graphs

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Abstract

There are many different mathematical objects (transitive reductions, minimal equivalent digraphs, minimal connected graphs, Hasse diagrams and so on) that are defined on graphs. Although they have different names they correspond to the same object if a weighted graph is defined more generally. The study of such generally defined graphs allows to consider some common properties of the objects, which seem different at the first glance.

This article presents a new kind of weighted graphs when the weights of the edges belong to a partially ordered set L. The case, when L is a positive linearly ordered monoid, is considered in more detail. For such L, the weight of a path is equal to the product of the weights of its edges. The exact lower bound of the weights of all paths between two vertices is the distance between these vertices. Graphs with the same set of vertices and equal distance for every pair of vertices form an equivalence class. One can define an order on the set of graphs in the natural way. It is shown that any equivalence class has a smallest element and a non-empty set of maximal elements, the bases. An algorithm is given to find a basis. When an equivalence class has only one basis is also shown.

1. Introduction

We consider finite oriented and non-oriented graphs without multiple edges or arcs; loops are admitted. Let the set of vertices of a graph be $V = \{1,2,\ldots,n\}$. A *path* in this graph is a sequence of (not necessarily distinct) vertices, $i_1 \cdots i_{k-1} i_k$, where for every $j = 1,\ldots,k-1$, (i_j, i_{j+1}) is an edge (or an arc in oriented case) of G. We assign to each path p an element $w(p)$ of a partially ordered set L, which is called the *weight* of the path p. Such a graph will be called a *weighted graph*, or, simply, a graph. A path from a vertex i to a vertex j is called an *(i,j)-path*. Let the set L have a greatest element ∞: $a \leqslant \infty$ for all $a \in L$.

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