Sharp estimates for integral means for three classes of domains

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Abstract

In this paper, the following sharp estimate is proved: $\int_0^{2\pi}|F'(e^{i\theta})|^p\,d\theta \leq \sqrt{\pi}2^{1+p}\Gamma(1/2+p/2)/\Gamma(1+p/2)$, $p>-1$, where F is the conformal mapping of the domain $D^- = \{\zeta:|\zeta| > 1\}$ onto the exterior of a convex curve, with $F'(\infty)=1$. For $p=1$, this result is due to Pólya and Shiffer. We also obtain several generalizations of this estimate under other geometric assumptions about the structure of the domain $F(D^-)$.

Keywords

analytic univalent function, conformal mapping, estimates for integral means, Euler beta function, harmonic function, Joukowski function