THE STRUCTURE OF THE KINETIC EQUATION FOR TIME CORRELATION FUNCTIONS

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The general kinetic equation for time correlation functions of statistical systems is constructed with the aid of four projection operators.

Earlier [1–3] the kinetic equation (KE) for time correlation functions)TCF) of statistical systems was found by the projection operators (PO) method. It has been shown recently [4] that the use of PO requires a consistent functional projection in two spaces: the initial functions and the values of the Liouville operator on four mutually supplementary PO. In this connection it is necessary to revaluate the structure of KE for arbitrary T C F.

Let us take the Liouville operator in matrix form

$$\hat{L} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}, \quad \begin{matrix} L_{11} = Q\hat{L}\Pi, & L_{12} = Q\hat{L}P, \\ L_{21} = K\hat{L}\Pi, & L_{22} = K\hat{L}P. \end{matrix} \qquad (1)$$

Here Π and $P = 1 - \Pi$ are PO in the space of functions, Q and $K = 1 - Q$ are PO in the space of values of the operator $\hat{L}$. They extract the irrelevant subspace connected with the initial TCF [2].

$$F(t) = \qquad (2)$$

$$\int \mathrm{d}q_N \, \mathrm{d}q_{N0} \, f_{N0} \, F^*(q_{m0}) \, F(q_m) \, W(q_N, t | q_{N0}, 0)$$

In (2) q_N is the phase point ($m < N$), $F(q_m)$ is the physical operator of m particles, f_{N0} the equilibrium initial distribution, W the conditional probability of transition form q_{N0} to q_N.

According to the method of [2,4–6] for composition KE, for the irrelevant part, connected with TCF $F(t)$, we obtain

$$\lambda \frac{\partial F(t)}{\partial t} = L_{11} \, F(t) + \int_\tau^t \mathrm{d}t' \, S(t, t') \, F(t') \qquad (3)$$

$$+ L_{12} \exp \left[(t-\tau) \, L_{22}\right] (1 - \Pi) \, F(\tau),$$

where λ is a constant determined by Q and the "collision operator" $S(t, t') = L_{12} \exp \{(t-t') L_{22}\} L_{21}$ is introduced, τ is the fixed time in the past evolution of the system. The operator $S(t, t')$ is also a TCF, but of a more complex form. Extracting the corresponding conditional probability, the Liouville operator and the matrix representation for this new function by analogy with (3), we find

$$\lambda_1 \frac{\partial S(t, t')}{\partial t} = L_{11}^{(1)} S(t, t') + \int_{t'}^{t} \mathrm{d}t'' S^{(1)}(t, t'') \, S(t'', t')$$

$$+ L_{12}^{(1)} \exp \{(t-t') \, L_{22}^{(1)}\} (1 - \Pi_1) S(t', t'). \qquad (4)$$

Here we have defined: $L_{11}^{(1)} = Q_1 \, \hat{L}^{(1)} \Pi_1$, $L_{12}^{(1)} = Q_1 \, \hat{L}^{(1)} P_1$, $L_{21}^{(1)} = K_1 \, \hat{L}^{(1)} \Pi_1$, $L_{22}^{(1)} = K_1 \, \hat{L}^{(1)} P_1$; the projector $\Pi_1 = R_1 G_1 = (L_{12} L_{21})^{-1} L_{21} \cdot L_{12}$ acts in the space of the functions $W_1(t | t') =$ $= \exp\{(t-t') L_{22}\} L_{21}$, PO Q_1 and $K_1 = 1 - Q_1$ in the space of values of contraction of the Liouville operator $\hat{L}^{(1)} = L_{22}$. We receive the KE for $S^{(1)}(t, t'')$ which is analogous to eqs. (3, 4) on the above mentioned procedure.

The basic difference between the systems of equations (3, 4) and the well known KE [1–3, 6] is in using four PO in each separate stage. It is of particular interest that the choice of PO Π and P (the operators in the Liouville operator functions space) is connected with the structure of the initial TCF. On the contrary PO in the $\hat{L}$ - values space may be taken arbitrary. This permits to find wide classes of solution for the TCF. Earlier only two PO were used (this means the assumption $\Pi \equiv Q$) and it essentially restricted the classes of possible solutions of the KE for the TCF.