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MATHEMATICAL LIFE

Farit Gabidinovich Avkhadiev (on his 70th birthday)

On 20 August 2017 Professor Farit Gabidinovich Avkhadiev, doctor of the physical and mathematical sciences, observed his 70th birthday.

He was born in a family of teachers in the village of Chembulat in the Kzyl-Yul district of the Tatar Autonomous Soviet Socialist Republic (now the Atnya district of the Republic of Tatarstan). In 1964 he enrolled in the Faculty of Mechanics and Mathematics of Kazan' State University, where he graduated in 1969 with distinction. Already during his student years Avkhadiev showed exceptional mathematical abilities, started doing research under the supervision of Professor L. A. Aksent'ev, and published his first papers. He recalls that after defending his Ph.D. thesis in 1972, it was not in his plans to become a professional scientist. He went to Algeria and taught mathematics there for several years. However, after returning to his country his interest in research grew stronger, and he proceeded to work in the Chebotarev Institute of Mathematics and Mechanics at Kazan' University, where he worked his way up from a junior researcher to the head of the Department of Mathematical Analysis and the Division of Mathematics of the institute. In 2008 he became the head of the Department of the Theory of Functions and Approximations in the Faculty of Mechanics and Mathematics at Kazan' State University.

Avkhadiev is a prominent expert in complex analysis and mathematical physics. He obtained brilliant results on sufficient conditions for univalence, inverse boundary value problems for analytic functions, and integral inequalities in Sobolev spaces for plane and space domains. His acute intelligence, rich imagination, scrupulosity in investigating problems, his ability to correctly formulate a problem and propose an optimal approach to it — all these qualities soon made him a highly respected authority in complex analysis.

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His achievements and results of his colleagues on sufficient conditions for univalence and related questions were presented in the surveys [1], [2], [5]–[7], [11] and the book [3]. Here we give a brief description of the most important and interesting parts of his research work.

He obtained many beautiful results related to the univalence of analytic functions. He found new sufficient conditions for univalence in terms of the Schwarzian derivative $(f''/f')' - (1/2)(f''/f')^2$ and pre-Schwarzian f''/f' of a function f. Several of his conditions for univalence and p-valence of analytic functions are stated in terms of their boundary behaviour. Results by M. Morse, F. D. Gakhov, and Yu. M. Krikunov on topological methods in the theory of functions of a complex variable influenced him to extend the scope of application of these conditions by considering more general classes of maps such as, for example, quasi-conformal mappings and then interior mappings in the sense of Stoilow, both in the plane and in the spaces $\mathbb{R}^n$.

The problem of constructing a Riemann surface over a sphere given the projection of its boundary is closely connected with these topics. It was stated by many famous mathematicians, such as Picard, Löwner, Hopf, and others. Avkhadiev (in conjunction with S. R. Nasyrov) obtained necessary and sufficient conditions for this problem to be solvable for boundary curves in a a fairly broad class. Together with Aksent'ev and G. G. Bil'chenko he found algorithms for constructing uni- and multivalent polygons with prescribed interior angles and applied these algorithms to the univalent solvability of inverse boundary-value problems and the analysis of the geometric properties of Hilbert boundary problems.

In the 1980s Avkhadiev developed a general approach to sufficient conditions for the uni- and p-valence of interior mappings in $\mathbb{R}^n$ in the sense of Stoilow, based on the concept of p-admissible functional. This approach was one of the main advances he made in his D.Sc. thesis (defended in 1990, at the Institute of Mathematics of the Siberian Branch of the USSR Academy of Sciences). In the early 1990s he and his student I. R. Kayumov turned to an investigation of Bloch spaces, and as a result, they solved the well known Anderson–Clunie–Pommerenke problem on finding estimates for the coefficients of the logarithms of derivatives in the class S of univalent functions on the disk. The many years of Avkhadiev's collaboration with his German colleague K.-J. Wirths were very fruitful: they obtained a number of new sharp estimates in Banach spaces of analytic functions, and in particular, they found sharp estimates that extend the classical principle of the hyperbolic metric to higher derivatives. They presented their results in the monograph [12].

Avkhadiev's research topics changed significantly from the mid-1990s: most of his results are now connected with various integral inequalities for functions in Sobolev spaces. The turning point was the paper [4], where he solved the generalized Saint-Venant problem of finding a geometric characteristic that would be equivalent to the torsional stiffness of elastic beams with simply connected cross-section. An analogue of Hardy's integral inequality can be derived from the inequalities in [4]. Avkhadiev's interests have been connected with this inequality and its generalizations in recent years. Here we present some of these striking results.

Let Ω be a proper open subset of $\mathbb{R}^n$, and let $\delta = \operatorname{dist}(x, \partial\Omega)$, $p \geqslant 1$, and $s > 1$. Denote by $c(p,s) = c_\Omega(p,s)$ the minimum possible constant in the integral inequality

$$\int_\Omega \frac{|f|^p}{\delta^s}\,dx \leqslant c(p,s)\int_\Omega \frac{|\nabla f|^2}{\delta^{s-p}}\,dx, \qquad f\in C_0^\infty(\Omega).$$

For $n=2$, Avkhadiev [8] found a 2-sided estimate for $c(p,2)$ in terms of a geometric characteristic of Ω which is easy to determine. For $n\geqslant 2$ he proved that if $s>n$, then the following sharp inequality holds for each p with $1\leqslant p<\infty$:

$$c_\Omega(p,s)\leqslant \frac{p}{s-p}\,.$$

This is a brilliant solution of a problem stated by J. L. Lewis (1988) and A. Wannebo (1990).

Another result [10] computes the sharp constant λ_ν in the inequality

$$\left(\frac{1}{4}-\nu^2\right)\int_\Omega \frac{|f|^2}{\delta^2}\,dx+\frac{\lambda_\nu^2}{\delta_0^2}\int_\Omega |f|^2\,dx\leqslant\int_\Omega |\nabla f|^2\,dx, \qquad f\in C_0^\infty(\Omega),$$

for a convex domain Ω in $\mathbb{R}^n$, where $\delta_0=\sup_\Omega \delta$ and $\nu\in[0,1/2]$ is a constant. The authors showed that λ_ν is the first positive root of Lamb's equation for Bessel functions. In particular, $\lambda_0=0.940$, which provides a sharp bound in a Hardy-type inequality proposed by H. Brezis and M. Marcus, and $\lambda_{1/2}=\pi/2$, which generalizes and refines the well-known isoperimetric inequalities due to Poincaré ($n=1$), Hersch ($n=2$), and Payne and Stakgold ($n\geqslant 3$).

It his work Avkhadiev has always devoted considerable attention to applied problems, and his analysis of them has provided him with new formulations of purely mathematical problems. In this connection we can point out his statement and solution (with D. V. Maklakov) of the inverse problem of recovering a hydrofoil section from a given cavitation diagram or the problem of maximizing the critical Mach number for aerofoil sections (together with A. M. Elizarov and D. A. Fokin).

Avkhadiev is an excellent lecturer and teacher. He presented results of his research in a clear and accessible form in his textbooks [9] and [14]. His students have defended six Ph.D. theses, and he has been the scientific advisor for two D.Sc. theses.

Mention should be made of the considerable time and effort he devotes to his position as the editor-in-chief of the journal *Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika*.[1]

Avkhadiev's interests outside mathematics are also very broad. He is interested in history, politics, and literature, and collects notable sayings of prominent scholars (some of these sayings decorate his office). Despite his numerous posts and duties, he finds time for outdoor activities. For many years he pursued whitewater sports together with friends and colleagues on catamarans, down swiftly flowing rivers in the Volga Region, the Urals, and Siberia. In some extreme situations during these trips Avkhadiev's best qualities became apparent: his calmness, self-confidence, openness, charm, and his philosophical attitude to life.

Farit Avkhadiev is a family man, a wonderful husband and father, selflessly devoted to his wife Nina Vasil'evna and daughter Kamila. He enjoys telling stories of his little grandson in Moscow, and is proud of his successes.

[1] Translated as *Russian Mathematics* (*Izvestiya VUZ. Matematika*).

From the bottom of our hearts we wish Farit Gabidinovich and his family every success, happiness, and good health.

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