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Thermodynamics of extremal rotating thin shells in an extremal BTZ spacetime and the extremal black hole entropy

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Abstract

© 2017 American Physical Society. In a (2+1)-dimensional spacetime with a negative cosmological constant, the thermodynamics and the entropy of an extremal rotating thin shell, i.e., an extremal rotating ring, are investigated. The outer and inner regions with respect to the shell are taken to be the Bañados-Teitelbom-Zanelli (BTZ) spacetime and the vacuum ground state anti-de Sitter spacetime, respectively. By applying the first law of thermodynamics to the extremal thin shell, one shows that the entropy of the shell is an arbitrary well-behaved function of the gravitational area A+ alone, S=S(A+). When the thin shell approaches its own gravitational radius r+ and turns into an extremal rotating BTZ black hole, it is found that the entropy of the spacetime remains such a function of A+, both when the local temperature of the shell at the gravitational radius is zero and nonzero. It is thus vindicated by this analysis that extremal black holes, here extremal BTZ black holes, have different properties from the corresponding nonextremal black holes, which have a definite entropy, the Bekenstein-Hawking entropy S(A+)=A+4G, where G is the gravitational constant. It is argued that for extremal black holes, in particular for extremal BTZ black holes, one should set 0≤S(A+)≤A+4G; i.e., the extremal black hole entropy has values in between zero and the maximum Bekenstein-Hawking entropy A+4G. Thus, rather than having just two entropies for extremal black holes, as previous results have debated, namely, 0 and A+4G, it is shown here that extremal black holes, in particular extremal BTZ black holes, may have a continuous range of entropies, limited by precisely those two entropies. Surely, the entropy that a particular extremal black hole picks must depend on past processes, notably on how it was formed. A remarkable relation between the third law of thermodynamics and the impossibility for a massive body to reach the velocity of light is also fo und. In addition, in the procedure, it becomes clear that there are two distinct angular velocities for the shell, the mechanical and thermodynamic angular velocities. We comment on the relationship between these two velocities. In passing, we clarify, for a static spacetime with a thermal shell, the meaning of the Tolman temperature formula at a generic radius and at the shell.

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