

# Arrows of times, non-integer operators, self-similar structures, zeta functions and Riemann hypothesis: A synthetic categorical approach

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## Abstract

© 2017 L & H Scientific Publishing, LLC. The authors have previously reported the existence of a morphism between the Riemann zeta function and the "Cole and Cole" canonical transfer functions observed in dielectric relaxation, electrochemistry, mechanics and electromagnetism. The link with self-similar structures has been addressed for a long time and likewise the discovered of the incompleteness which may be attached to any dynamics controlled by non-integer derivative operators. Furthermore it was already shown that the Riemann Hypothesis can be associated with a transition of an order parameter given by the geometric phase attached to the fractional operators. The aim of this note is to show that all these properties have a generic basis in category theory. The highlighting of the incompleteness of non-integer operators considered as critical by some authors is relevant, but the use of the morphism with zeta function reduces the operational impact of this issue without limited its epistemological consequences.

<http://dx.doi.org/10.5890/JAND.2017.06.012>

## Keywords

Fractal geometry, Non integer operators, Riemann hypothesis, Zeta function

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