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Quadrature Finite Element Method for Elliptic Eigenvalue Problems

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Abstract—A positive semi-definite eigenvalue problem for second-order self-adjoint elliptic differential operator defined on a bounded domain in the plane with smooth boundary and Dirichlet boundary condition is considered. This problem has a nondecreasing sequence of positive eigenvalues of finite multiplicity with a limit point at infinity. To the sequence of eigenvalues, there corresponds an orthonormal system of eigenfunctions. The original differential eigenvalue problem is approximated by the finite element method with numerical integration and Lagrange curved triangular finite elements of arbitrary order. Error estimates for approximate eigenvalues and eigenfunctions are established.

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1. INTRODUCTION

Differential eigenvalue problems have important applications in mechanics and physics [1–16]. Finite element approximations of differential eigenvalue problems leads to the matrix eigenvalue problem with the matrices whose elements involve integrals which are usually evaluated by numerical quadrature formulas. The effect of numerical integration on the eigenvalue and eigenfunction errors has been investigated in the papers [17–35]. In the papers [30–33] error estimates for approximate eigenvalues and eigenelements of eigenvalue problems in a Hilbert space have been derived. In the present paper we apply these results for investigating the finite element approximation of the following positive semi-definite elliptic differential eigenvalue problem

$$-\sum_{i,j=1}^{2} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial u(x)}{\partial x_j}\right) + a_0(x)u(x) = \lambda b_0(x)u(x), \quad x \in \Omega, \tag{1}$$

$$u(x) = 0, \quad x \in \Gamma. \tag{2}$$

Here Ω is a plane domain with smooth boundary Γ, $\overline{\Omega} = \Omega \cup \Gamma$, the coefficients $a_{11}(x)$, $a_{22}(x)$, $a_{12}(x) = a_{21}(x)$, $a_0(x)$, $b_0(x)$, $x \in \overline{\Omega}$, are smooth functions, for which there exist positive constants $\widetilde{\alpha}_1$, $\widetilde{\alpha}_2$, $\widetilde{\beta}_2$, such that

$$\widetilde{\alpha}_1 \sum_{i=1}^{2} \xi_i^2 \le \sum_{i,j=1}^{2} a_{ij}(x)\xi_i\xi_j \le \widetilde{\alpha}_2 \sum_{i=1}^{2} \xi_i^2 \quad \forall \xi_1, \xi_2 \in \mathbb{R},$$

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